7

ESTUARIAL MODELS

Gerald T. Orlob

President, Resource Management Associates
Lafayette, California

Professor of Civil Engineering
University of California, Davis

7-1 THE PROBLEM

The estuary is without a doubt the most complex and, therefore, the most challenging system with which the water-resources analyst must deal. Natural irregularities in geometry, inherent unsteadiness in flows, and the myriad activities of man tend to make the estuary a prime subject for investigation. Adding the dimensions of quality, i.e., physical, chemical, and biological actions and interactions, puts the estuarial system at the head of the list of formidable topics for investigation. Many millions of dollars have been allocated by governmental agencies to the collection of information on estuarial behavior, the design and operation of physical hydraulic models of estuaries, and the development of programs of estuarial management.

Predicting the behavior of the estuarial system has challenged the ingenuity of a host of investigators over the years. Until recently, however, the complexities of the system's behavior, although stimulating to the analytically minded, have generally defied practical analytical solution. Physical hydraulic models have been used in some cases to describe effects of physical changes, but they have not been able to deal with the dimension of quality. The recent concern for the total environment, and that of the aquatic ecosystem in particular, has expanded the quality dimension well beyond the potential reach of physical models. Only with the advent of modern computational capability has it become feasible to seek and find analytical solutions to this new and formidable class of problems. Mathematical models of estuarial systems, solvable on digital computers, have become the most useful tools for predicting the consequences of future actions in estuarial water management.

In this chapter we will describe some basic theoretical considerations upon which most of the current models are founded, considering both quantity and quality relationships and introducing some new concepts of ecological modeling as applied to estuaries. Next, we will briefly review the history of estuarial mathematical model development, summarizing the work of four major groups of investigators in the United States and showing the scope of practical application of models developed by these workers. To place the whole matter of modeling in pragmatic perspective, a few case studies using these models will be reviewed.

7-2 THEORETICAL CONSIDERATIONS

7-2.1 Quantity versus Quality

The general thrust of estuarial investigation in the United States over the postwar period through the early 1960s was toward solution of quality problems. Because the bulk of this work was originated or sponsored by the U.S. Public Health Service, Division of Water Supply and Pollution Control, it is perhaps understandable that the initial emphasis was placed on quality, per se. Quantity considerations, i.e., hydrodynamic behavior of the estuarial systems, were almost ignored. In studies of stream systems it has been customary to assume steady flow, complete mixing, and simple dilution; these assumptions were at first simply extended to the estuarial problem. Attention was focused on the kinetics of reaeration, biodegradation, and the like, as in the "classic" case of the "oxygen sag" in stream systems; hydrodynamics seemed not to be the concern of the sanitary engineer.

Perhaps this situation prevailed for want of sound hydrodynamic solutions upon which to base the quality assessment or for lack of suitable inspiration in terms of a real need. Whatever the reason for not bringing together hydrodynamic and quality considerations up to this time, the advent of intensified interest in estuarial quality problems provided the necessary stimulus to move ahead. The most important practical examples are the comprehensive studies initiated by the USPHS of the estuaries of the Delaware River on the eastern seaboard[1,2] and the Sacramento–San Joaquin River Delta in California.[3]

In both these cases there was an awareness of the coupling of quantity and quality, at least of the strong dependence of quality on hydrodynamic (or hydrologic) behavior. Both groups of investigators described quality changes in the estuarial system through application of the so-called *diffusion equation*. This equation and its companion *mass conservation equation* form the foundation of the majority of water-quality models in use today. A review of the concepts embodied in the equation provides essential background for the more detailed theoretical considerations of estuarial hydrodynamics and quality.

7-2.2 Advection-Effective Diffusion

Let us consider for the present that we are dealing only with the mass transport of a conservative substance, say, the water itself or some solute such as ocean salinity that is neither created nor destroyed within the system we are considering. The distribution of this substance in space and over time in a natural water environment may be accomplished by several phenomenological mechanisms, including convective displacement, often termed simply "plug flow" or advection, molecular diffusion, turbulent diffusion, and so-called eddy diffusion. By analogy to the random motions of molecules in Brownian motion, other random processes, such as those associated with pure fluid turbulence, can be described for a fluid body by the Fickian diffusion equation

$$\frac{\partial C}{\partial t} = -\nabla \cdot CU_i + \nabla \cdot (D_m \nabla C) + \nabla \cdot (\epsilon_i \nabla C) \qquad (7\text{-}1)$$

where C is the concentration of a conservative substance identified with the fluid molecules, D_m is molecular diffusivity, ϵ_i is turbulent diffusivity, U_i is a vector velocity, and the subscript i denotes the vector direction. It is generally acknowledged that for natural systems such as estuaries, $D_m \ll \epsilon_i$. In fact, D_m may be neglected entirely at the scales and times associated with the mixing processes of greatest interest.

The scales of the mixing phenomena themselves may be of particular importance in extending the analogy to describe the entire transport process by an equation of the form of Eq. (7-1). If one chooses a scale for model representation (either in time or space) that is large compared to the temporal or spatial variations in U_i, or if ϵ_i is highly variable, then the analogy breaks down. However, to the degree that such variations may be regarded as the same statistical character as molecular motion, it may be convenient to represent the total advection-mixing process by an equation of the form

$$\frac{\partial C}{\partial t} = -U_i \nabla \cdot C + \nabla \cdot (\mathbf{E}_i \nabla C) \qquad (7\text{-}2)$$

where $\mathbf{E}$ is the vector "effective diffusivity" describing all mixing mechanisms, i.e.,

$$\mathbf{E}_i = D_m + \epsilon_i + \mathbf{E}_{ai} \qquad (7\text{-}3)$$

where $\mathbf{E}_{ai}$ represents all other random mixing processes. The degree to which such factors as advective dispersion, wind induced mixing, tidal oscillations, and the like, may in the aggregate be treated in a practical sense as random processes, when such is truly not the case, is the test of the veracity of the analogy and the model that derives from it.

It follows from this argument that if the scale at which we can view the mixing process is close to the scale of pure turbulence, we may use the equation to construct a fairly reliable model of mass transport processes. Unfortunately, very small scales are not always practical in model development for reasons of cost, insufficiency of data, or imperfect knowledge of the prototype. In such cases we may elect to depend more on the diffusional analogy, i.e., emphasize the second term in the right-hand member of Eq. (7-2). How much reliance is placed on this term and how much effort is expended to improve the description of advective processes or other pseudorandom processes are what most distinguish the various estuarial models that have been developed.

The dominant mass transport mechanisms in estuarial systems are usually in the horizontal plane. Vertical transport, although not negligible, is usually considered as relatively small in proportion to other transport terms. In shallow systems, or those that are strongly affected by wind and wave action, transport along the vertical axis is often considered to be of negligible consequence in describing either the hydrodynamic behavior or the disposition of quality constituents. Such estuaries, a large and important class, are often characterized as "vertically mixed." They are treated in the modeling exercise as two-dimensional.

Equation (7-2) may be written for the two-dimensional case as

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} - v\frac{\partial C}{\partial y} + \frac{\partial}{\partial x}\left[E_x(t)\,\frac{\partial C}{\partial x}\right] + \frac{\partial}{\partial y}\left[E_y(t)\,\frac{\partial C}{\partial y}\right] \qquad (7\text{-}4)$$

where u and v are the instantaneous tidal velocities in the x and y directions, respectively, and $E_x(t)$ and $E_y(t)$ are the corresponding effective diffusion coefficients defined as in Eq. (7-3) to include all mixing effects.

As noted previously, the scale of the phenomenon, which is a measure of how well we choose to describe certain transport mechanisms, determines the magnitude and character of the effective diffusion coefficients. If u and v are approximated, for example, by the time-averaged velocities $\bar{u}$ and $\bar{v}$, where they actually fluctuate appreciably about the mean values, then it will be necessary to increase the corresponding values of $E_x(t)$ and $E_y(t)$ to compensate. It would be assumed, of course, since "diffusion" is by analogy a random phenomenon, that the velocity fluctuations are also random. To the extent they are actually periodic instead, as in the case of time-averaged oscillations of a tidal current, the analogy is less than satisfactory. Likewise, spatial averaging which includes phenomena that could be described by more complete hydrodynamic analysis will necessitate upward adjustments in the coefficients. An example is the case of a tidal channel with very rough banks for which the velocity is considered constant bank to bank.

It will be necessary in the numerical solution to Eq. (7-4) to utilize some empirical expression for the effective diffusion coefficients or to derive such coefficients from field experience. As a practical matter, there is no completely satisfactory way to avoid dependence on some empiricism, only to minimize it to an acceptable level. One approach suggested by the form of the equation itself is to improve the description of the hydrodynamic phenomena, i.e., the first two terms on the right side of Eq. (7-4). This requires solution of the hydrodynamic equations of motion and continuity.

7-2.3 Hydrodynamics of Tidal Motion

The dominant motions in shallow estuarial systems are those in the horizontal plane, i.e., along the x and y axes. Moreover, the greater number of cases of interest in the United States from the quality viewpoint are relatively shallow estuaries, systems within which wind and wave action preclude stratification. Consequently, the present state-of-the-art emphasizes solutions of estuarial hydrodynamics that are essentially two-dimensional (in the mathematical sense).

Three hydrodynamic equations are required to describe the flow regime in a shallow, vertically mixed estuary—two equations of motion, and one of continuity.[4] Following the notation of Masch and Brandes,[5] these equations are

Motion in the x direction:

$$\frac{\partial q_x}{\partial t} = -gd\frac{\partial h}{\partial x} - gd\, S_{ex} + X_w - \Omega q_y \tag{7-5}$$

Motion in the y direction:

$$\frac{\partial q_y}{\partial t} = -gd\frac{\partial h}{\partial y} - gd\, S_{ey} + Y_w + \Omega q_x \tag{7-6}$$

Continuity:

$$\frac{\partial h}{\partial t} = S - \frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial v} \tag{7-7}$$

where q_x, q_y = tidal flows per unit width along x and y axes, respectively
h = elevation of tidal water surface above reference datum
d = depth of water = $h - z$
z = elevation of bottom above reference datum
t = time
g = acceleration of gravity
Ω = Coriolis parameter
X_w, Y_w = wind stresses per unit density of the water along the x and y axes, respectively
S = net rate of accretion or extraction from the element, due to rainfall, evaporation, and extraneous import or export
S_{ex}, S_{ey} = energy gradients along the x and y axes, respectively

Equations (7-5) and (7-6) neglect the Bernoulli acceleration terms that have been found to be negligible for shallow embayments with small to medium tidal ranges.

The energy gradients may be obtained from any of a number of open-channel flow equations, the most convenient of which is probably the Manning equation

$$q = \frac{1.49}{n} d\, R^{2/3} S_e^{1/2} \tag{7-8}$$

where n is Manning's coefficient, a function of the absolute roughness of the channel bottom, and R is the hydraulic radius, approximately equal to the depth d for wide shallow channels. Equation (7-7) may be rewritten for the two coordinate directions as

$$S_{ex} = \frac{n^2}{2.21\, d^{10/3}} q q_x \tag{7-9}$$

and

$$S_{ey} = \frac{n^2}{2.21\, d^{10/3}} q q_y \tag{7-10}$$

in which

$$q = \sqrt{q_x^2 + q_y^2} \tag{7-11}$$

It will be noted that q is always positive, although q_x and q_y may be positive or negative; i.e., frictional resistance is always opposed to the direction of flow.

Wind stresses may be introduced into Eqs. (7-5) and (7-6) in accordance with the relations

$$X_w = \frac{\tau_w}{\rho} \cos\theta = k_w V_w^2 \cos\theta \tag{7-12}$$

$$Y_w = \frac{\tau_w}{\rho} \sin\theta = k_w V_w^2 \sin\theta \tag{7-13}$$

where τ_w is the wind stress on the water surface, k_w is a wind stress coefficient, V_w is the wind velocity at a fixed elevation above the water surface, and θ is the angle of the wind direction from the x axis. Empirical expressions for estimating k_w have been developed.[5]

Coriolis acceleration can be important in open ocean systems or in large embayments, particularly those of roughly circular shape with equivalent diameters of the order of 30 km or greater. The effect of the Coriolis force is to deflect the current to the right in the direction of flow in the Northern Hemisphere. The parameter Ω is computed from

$$\Omega = 2\omega \sin\theta \tag{7-14}$$

where ω is the angular rotation of the earth ($\omega \cong 7.27 \times 10^{-5}$ rad/sec) and ϕ is the latitude.

7-2.4 Quality Considerations

From the viewpoint of practical numerical solution of estuarial quality problems a more convenient form of the advection-effective diffusion relation is the mass conservation equation

$$\frac{\partial M}{\partial t} = -uC - vC + E_x \frac{\partial C}{\partial x} + E_y \frac{\partial C}{\partial y} \pm s \qquad (7\text{-}15)$$

where M = mass of constituent per unit volume
s = sources and sinks of constituent
and the other terms are as previously defined.

For the two-dimensional, shallow estuary the rate of mass transfer of the constituent to or from a water column of depth d is

$$\frac{\partial (dC)}{\partial t} = -q_x C - q_y C + E_x d \frac{\partial C}{\partial x} + E_y d \frac{\partial C}{\partial y} \pm s \qquad (7\text{-}16)$$

where q_x, q_y are the tidal flows per unit width along the x and y axes, respectively.

As written, the equation becomes the basis for any system of interactive quality relationships that can be characterized by first-order kinetics.

Appropriate first-order terms are merely substituted for the source-sink parameter s to convert the equation to describe a "nonconservative" substance. In this way, convenient mathematical expressions may be derived for such simple reactions as biodegradation and bacteria die-away and for dual interactive reactions such as the classic treatment of dissolved oxygen and biochemical oxygen demand (Streeter-Phelps). Complex interactions involving many parameters, such as the ecologic succession from nutrients through bacteria, algae, zooplankton and to fish present some special problems but the approach appears promising. Limitations lie primarily in description of reaction kinetics for real systems and in providing a realistic hydrodynamic description of the systems behavior.

These advances in the estuarial modeling art are represented by four major developments covering the decade 1962–1972:

Delaware Estuary Comprehensive Study, leading to the creation of the so-called DECS model by Thomann and his coworkers
Bay-Delta Models, formulated and applied by Orlob, Shubinski and their associates
Gulf Coast Estuary Model, developed by Masch and his associates
Rand Estuary Model, developed by Leendertse

In addition, two extensions of the Bay-Delta Models may be cited as unique extensions of the state-of-the-art of estuarial modeling:

Triangular Element Model, developed and applied by Shubinski
An Estuary Ecologic Model, developed by Chen

These developments are briefly discussed herein.

7-2.5 The Delaware Estuary Model

Thomann and his associates[2,6,7,8] developed one of the best known and perhaps most extensively applied estuarial models, that of the Delaware River Estuary. The model, developed under sponsorship of the U.S. Public Health Service in its Delaware Estuary Comprehensive Study (DECS), was conceived as a tool to describe the seasonal steady-state oxygen concentrations within a well-mixed estuary. It is referred to as the DECS model, after the study in which it was developed.

Conceptually, the DECS model treats the estuary as a linear system of 30 discrete segments within which the flow is regarded as unidirectional and steady in accordance with the net hydrologic flux. Segments vary in length from 2 to 4 miles, averaging about 2.8 miles, and in the aggregate span a prototype distance of 83 miles below Trenton, New Jersey. This system of elements is shown in its relation to the prototype estuary in Fig. 7-1. Figure 7-2 is a definition sketch.

Some basic assumptions inherent in the DECS Model are

1 Flow is steady for the season of interest.
2 Velocities are unidirectional in accordance with net hydrologic balance for each segment.
3 Mixing due to tidal effects or other unsteadiness in the flow may be considered as random, i.e., diffusional.
4 Complete mixing is assumed for each segment.

A basic mass balance equation for oxygen in a particular segment i of volume V_i may be formulated as

$$\begin{aligned} V_i \frac{dC}{dt} = {} & Q_{i-1,i}\,[\xi_{i-1,i}\,C_{i-1} + (1-\xi_{i-1,i})C_i] \\ & - Q_{i,i+1}\,[\xi_{i,i+1}\,C_i + (1-\xi_{i,k+1})\,C_{i+1}] \\ & + B_{i-1,i}\,(C_{i-1} - C_i) - B_{i,i+1}\,(C_i - C_{i+1}) \\ & + r_i V_i (C_{si} - C_i) - K_i V_i L_i \pm S_i \end{aligned} \qquad (7\text{-}17)$$

where $i-1,\ i,\ i+1$ = successive segments
V = volume of segment
Q = flow
C = dissolved oxygen concentration
C_s = dissolved oxygen saturation concentration
r = reaeration rate
K = biochemical decay rate
L = ultimate biochemical oxygen demand
S = extraneous sources and sinks of oxygen

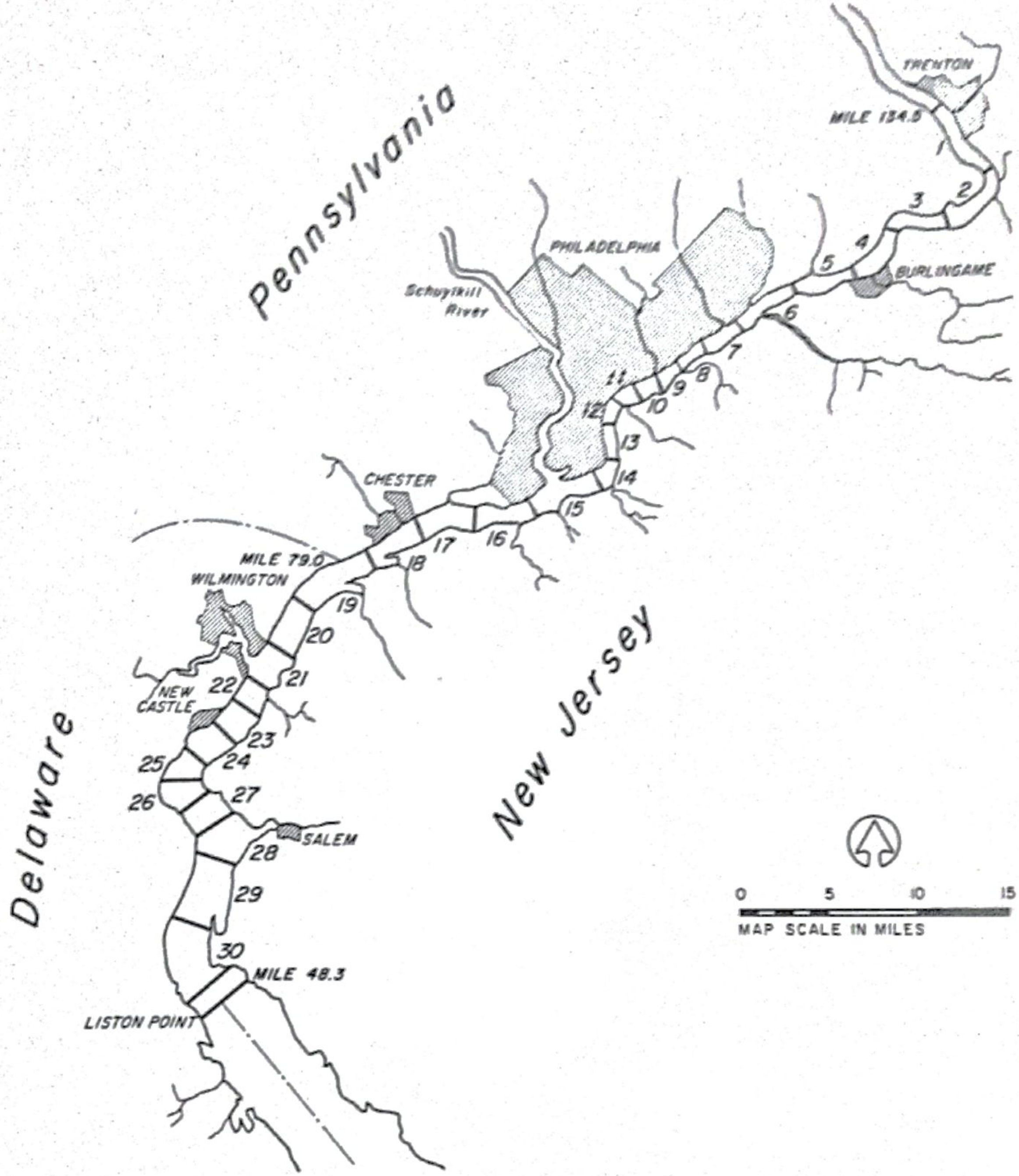

FIGURE 7-1
Delaware River estuary, showing DECS Model configuration.[1]

ξ = tidal exchange coefficient ($\xi = 0.5$ for pure tidal oscillation with no net hydrologic flux and $\xi = 1.0$ for stream condition with no tidal effects)

$B = AE/\bar{X}$, in which A is the cross-sectional area, E is "effective diffusion," and $\bar{X}$ is the sum of the half segment lengths on either side of the cross section considered

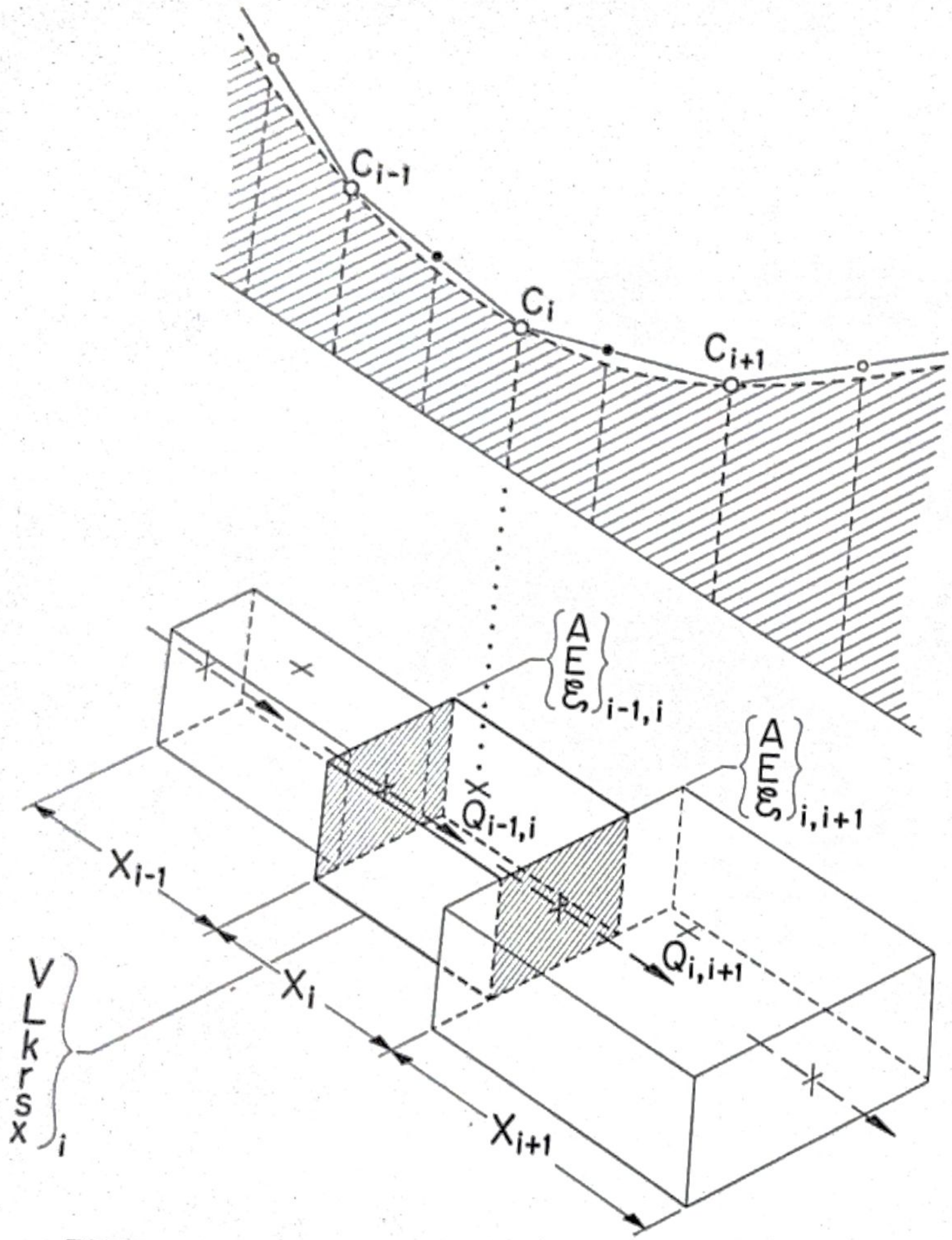

FIGURE 7-2
Definition sketch—finite-segment estuarial model.

Since, in Eq. (7-17), both the oxygen concentrations C_i and the biochemical oxygen demands L_i are unknown, it is necessary for solution to write a second equation for the segment. The rate of change of the oxygen-consuming loads in a segment i of volume V_i is

$$V_i \frac{dL_i}{dt} = Q_{i-1,i}\,[\xi_{i-1,i}L_{i-1} + (1-\xi_{i-1,i})L_i] - Q_{i,i+1}\,[\xi_{i,i+1}L_i + (1-\xi_{i,i+1})L_{i+1}] + B_{i-1,i}\,(L_{i-1} - L_i) - B_{i,i+1}\,(L_i - L_{i+1}) - r_i V_i L_i \pm J_i \qquad (7\text{-}18)$$

where J_i are sources of extraneous BOD load to the segment, i.e., waste discharges, and other terms are as previously defined.

Given the geometric properties of the system (V, A, and x), the net flows (Q), the BOD loads (L and J), and the various coefficients (ξ, B, C_s, r, K, and E), it is now possible to write two equations for each segment, i.e., two sets of equations for the system. These comprise the mathematical model of the estuary as formulated for the Delaware. Solution for the DECS Model is obtained by matrix algebra involving three transformations: effluent BOD to stream BOD, stream BOD to stream DO, and finally the combination effluent BOD to stream DO. As illustrated in Fig. 7-2 the model gives a steady-state profile of concentration along the principal axis of the estuary.

The DECS Model has been extensively applied to the Delaware estuary in the form described above by the Federal Water Pollution Control Administration, that agency's successors, and the Delaware River Commission. It has been extended by Thomann et al.[9] to include prediction of the fate of nitrogen (organic, ammonia, nitrite, and nitrate) in the estuary in what is termed a "feed-forward multistage" system.

Other applications of the DECS Model include the Potomac River[10,11,12] and Hillsborough Bay in Florida.[13] In the latter case the model was adapted to the two-dimensional problem utilizing an interdependent system of 38 segments.

7-2.6 The Bay-Delta Models

The Bay Delta Models were developed over a period beginning in late 1962 and extending through 1968. Initial modeling work was undertaken by the author and his coworkers, under the sponsorship of the U.S. Public Health Service; later it was supported by FWPCA (FWQA, EPA) and the state of California in connection with comprehensive studies of the Sacramento–San Joaquin Delta and San Francisco Bay.[3,14,21] The models have been extensively applied by Water Resources Engineers, Inc., and various state and federal agencies to an unusually diverse array of estuaries and estuarial problems. They have provided information on estuarial behavior that has been directly used in water-resource planning and management. Consequently, in their present state the models are proven tools of considerable utility to water-resource planners for certain classes of estuarial problems.

Estuarial Hydrodynamic Model. Simulates tides, currents, and discharges in shallow, vertically mixed estuaries excited by ocean tides, hydrologic influx, and wind action.

Dynamic Water Quality Model. Simulates the mass transport of either conservative or nonconservative quality constituents utilizing information derived from the hydrodynamic model.

Steady-State Water Quality Model. Predicts steady-state distributions of conservative and nonconservative quality constituents utilizing net tidal flows derived by integration of the output of the hydrodynamic model.

Each of the three models is structured conceptually for a particular application as a one-dimensional network approximation of a shallow, fully mixed system of interconnecting channels and embayments. The adaptation of the network to the prototype configuration is illustrated by Figs. 7-3 and 7-4, the latter showing the flexibility afforded the user in providing more detail in areas of special interest.

The network scheme used in the early Bay-Delta Models was a natural result of initial efforts to solve the hydrodynamic problems of the Sacramento–San Joaquin Delta, an actual system of discrete channels. A distinguishing feature of the Delta was the fact that these channels were interconnected, i.e., branched. Moreover, flows within the system were known to be strongly influenced by pumping within the Delta and upstream regulation of inflow, factors that contributed to seasonal reversals in net flow. Coupled with strong tidal and wind influences on the Delta channels and the connecting embayments, these effects made prediction of flows, either steady or dynamic, a virtual impossibility without a model.

The basic approach for application of the Bay-Delta Models is to discretize the system, much in the same manner as for the DECS Model, except that branching and looping is permitted. In places where channels broaden into embayments, a network of one-dimensional elements can be constructed, approximating the volumetric and flow effects on the larger system. In this way the model is formed as a system of volumetric units, called *nodes*, comprising the half volumes of connecting *links* as illustrated in the definition sketch of Fig. 7-5.

Typically, nodes can be of any polygonal shape and node centers can be spaced by the modeler as required by the problem being studied and the estuary's geometry. Nodes have characteristics of volume, surface area, depth, and surface elevation. In the quality models, first-order coefficients such as reaeration decay, growth, etc., are assigned by node as required for the particular problem. Quality constituents, both concentrations and mass are identified with nodes.

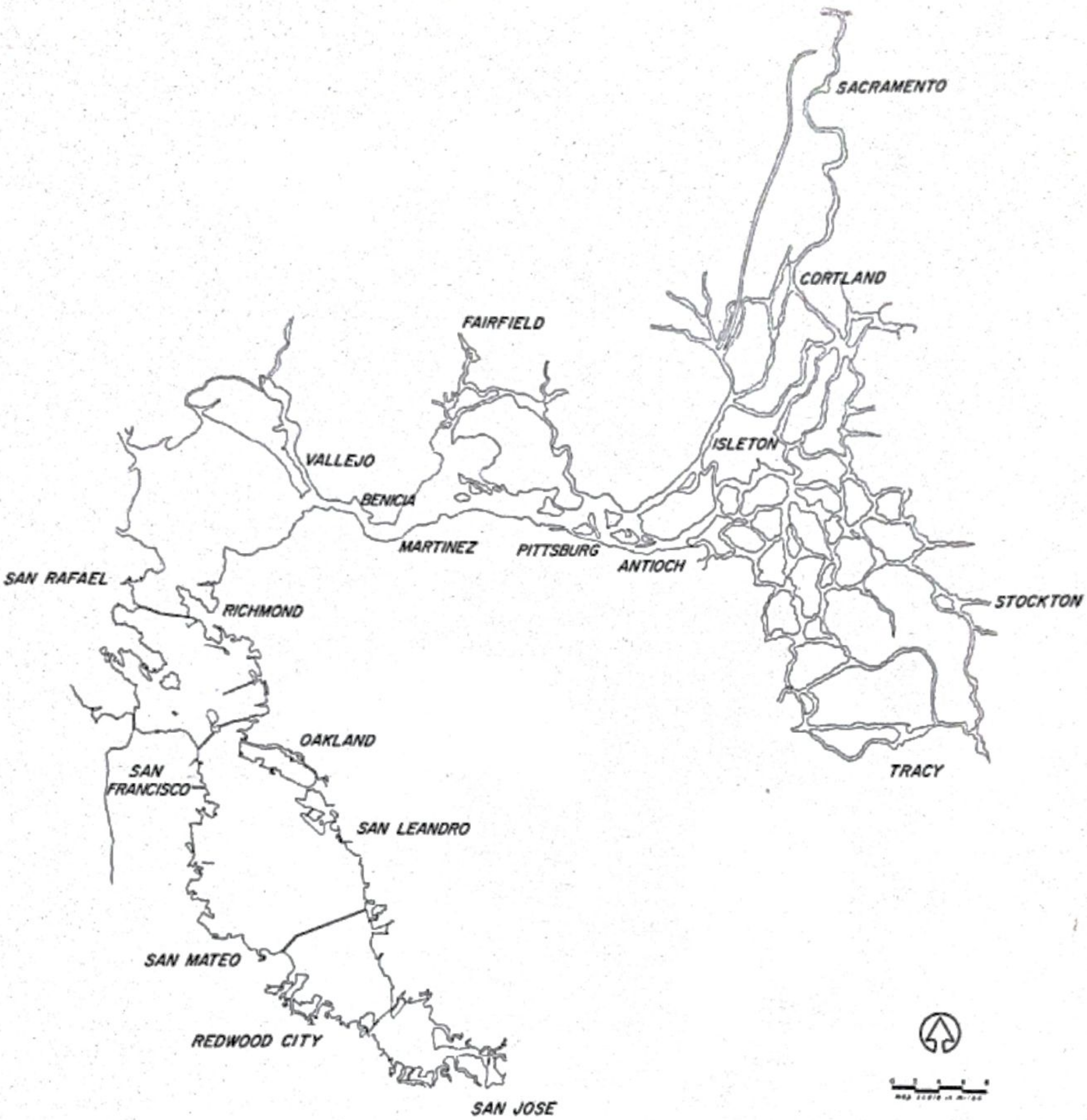

FIGURE 7-3
San Francisco Bay-Delta estuary.

Flows in the Bay-Delta Models are envisioned to occur between nodes along connecting links that represent actual estuarial channels or portions of an embayment. Each link is assumed to have the properties of a broad open channel, i.e., length, depth (hydraulic radius), surface width, cross section, and roughness. In addition, for quality simulation effective diffusion coefficients can be assigned to or computed for each link in the system.

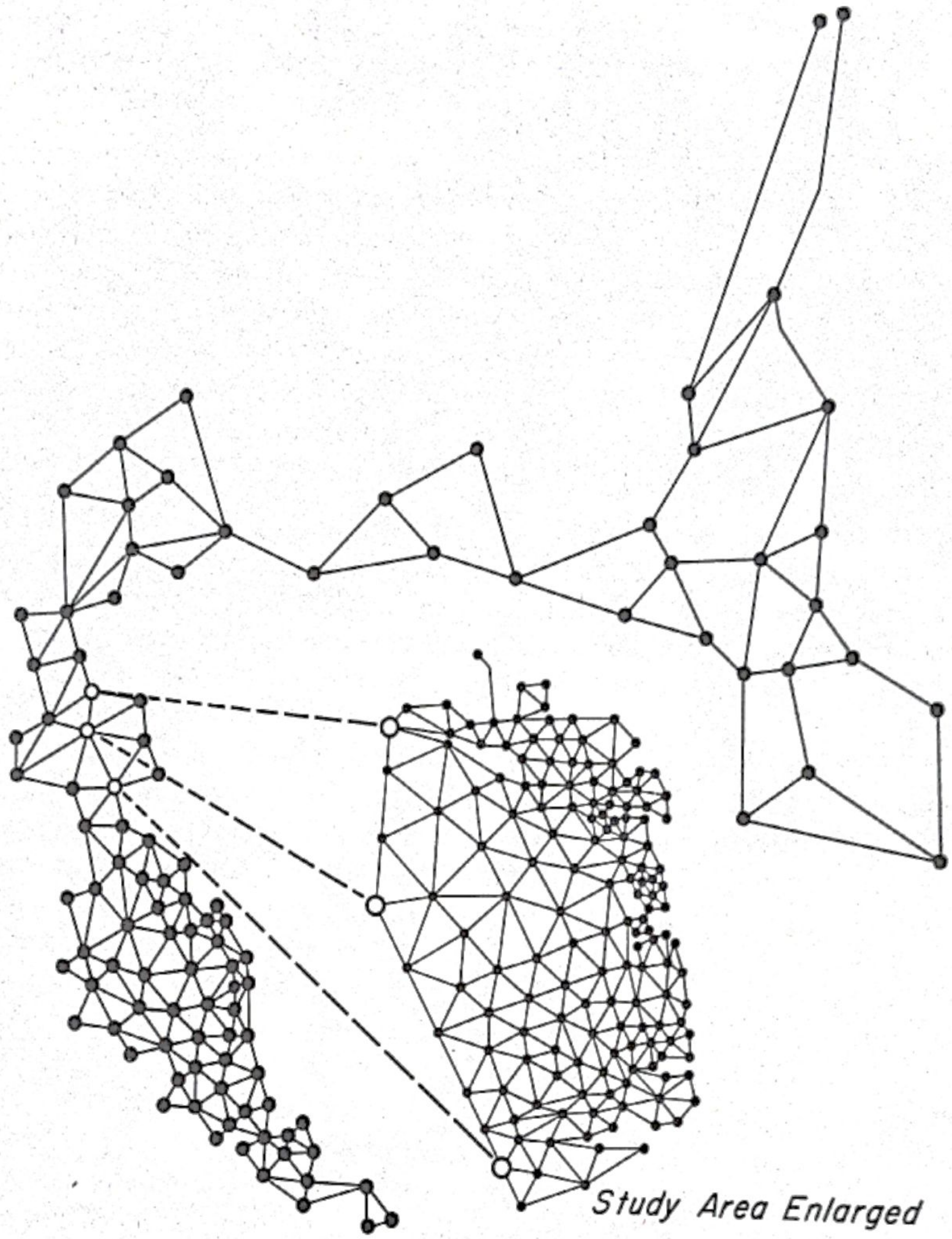

FIGURE 7-4
Bay-delta estuarial mathematical model network.

7-2.7 Estuarial Hydrodynamic Model

The Estuarial Hydrodynamic Model[15,16] solves a set of one-dimensional equations of motion written for links comprising the estuarial network and a set of continuity equations for the nodes. In the general notation used previously, these are

Equation of motion (along a link):

$$U_i^t = U_i^{t-1} - \left(U^{t-1} \frac{\Delta U}{l_i} + g \frac{\Delta h}{l_i} + K|U^{t-1}|U^{t-1} \right) \Delta t \qquad (7\text{-}19)$$

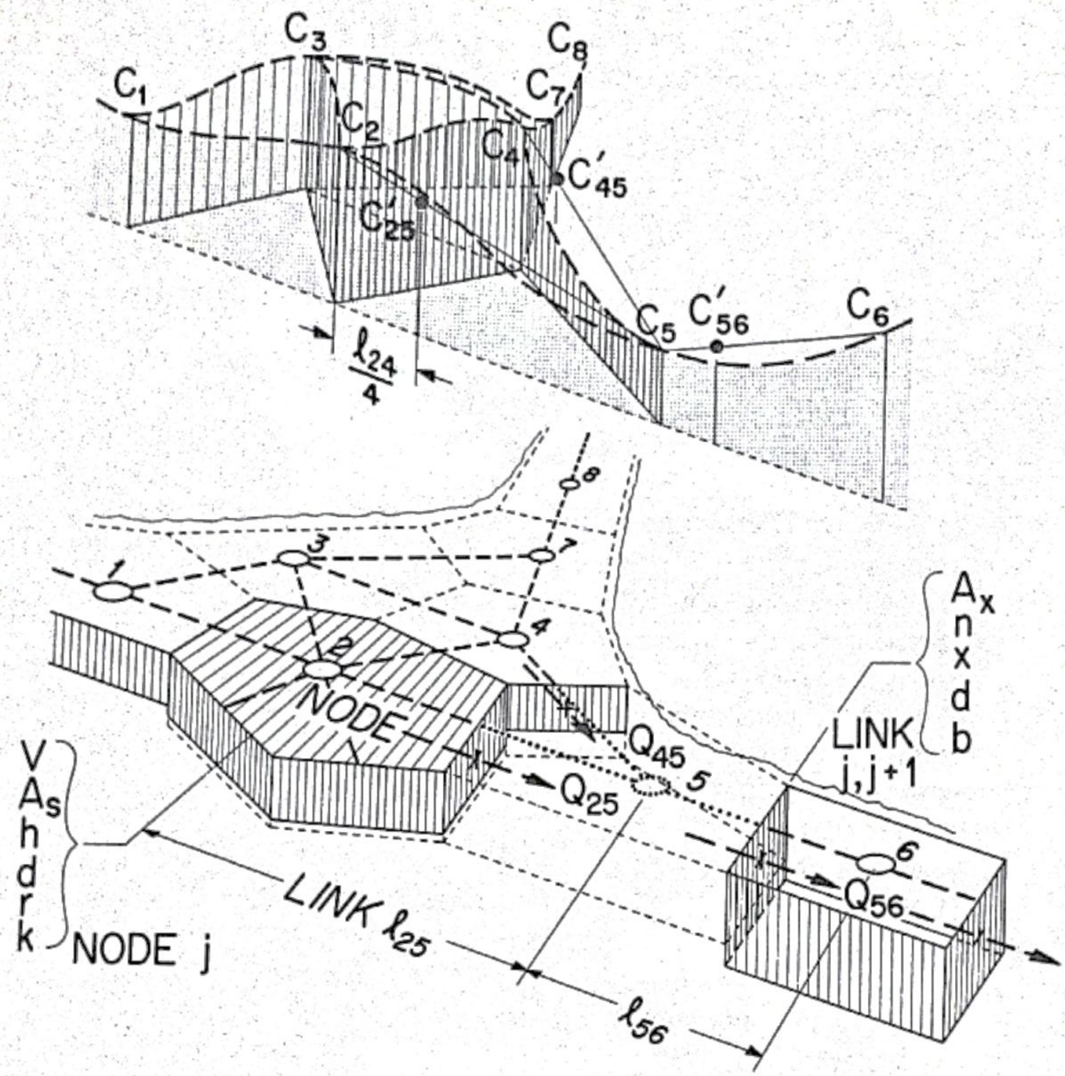

FIGURE 7-5
Definition sketch—WRE node-link estuarial model.

where U_i^t = average velocity in link i at time t

U_i^{t-1} = average velocity for previous time, $t-1$

ΔU = difference in average velocity at time t over length of link l_i, computed as the difference in weighted averages of velocities in links connecting to nodes j and $j+1$ at either end of link i

l_i = length of link

g = acceleration of gravity

$\Delta h = h_j - h_{j+1}$ = difference in water surface elevation between nodes j and $j+1$

$K = gn^2/2.21\ d^{4/3}$ = frictional resistance coefficient

n = Manning's n

d = average channel depth in link i (≅ hydraulic radius for broad shallow channels), adjusted on the basis of changes in h_j and h_{j+1} and flow continuity in link l

Δt = time interval

Equation of continuity (at a node):

$$h_j^t = h_j^{t-1} - \frac{\Sigma Q_j}{A_{sj}} \Delta t \tag{7-20}$$

where h_j^t = water surface elevation at node j at time t

h_j^{t-1} = water surface elevation at time $t - 1$

ΣQ_j = net flow from or to node j

A_{sj} = surface area of node j

The solution technique for the Estuarial Hydrodynamic Model is a step-forward explicit procedure with the following steps:

Initialization:

1 Set inflow conditions for nodes at landward boundaries, $q_j = f(t)$.

2 Set water surface elevations or tidal conditions for nodes at seaward boundaries, $h_j = f(t)$.

3 Set extraneous inflow-outflow conditions for all interior nodes, including net flows from import, export, waste effluents, and evaporation.

Solution sequence:

1 Calculate link flows Q_l^t and velocities U_l^t based on continuity.

2 Calculate weighted average velocities, $\bar{U}_j^t$ and $\bar{U}_{j+1}^t$, of channels entering or leaving nodes.

3 Calculate $\Delta U^t = \bar{U}_j - \bar{U}_{j+1}$.

4 Estimate Δh and $\Delta h/l$ at half step.

5 Calculate $U_l^{t+1/2}$ from Eq. (7-19) for $\Delta t/2$.

6 Calculate ΔU and $\Delta U/l$ at half step.

7 Calculate U_l^{t+1} from Eq. (7-19) for Δt.

8 Calculate link flows, Q_l^{t+1}.

9 Repeat.

The solution results in spatial and temporal descriptions of velocities and flows in links and water surface elevations in nodes.

A necessary condition for a stable solution using the explicit procedure outlined above is that for all links

$$\Delta t \leq \frac{l_l}{c} \tag{7-21}$$

where Δt = maximum time step for a stable solution

l_l = length of link

c = celerity of shallow water wave disturbance $\cong \sqrt{gd}$

The customary practice in applying the models is to impose a representative set of hydrologic and tidal conditions at the boundaries and run the model over several cycles, say, three diurnal periods, until hydrodynamic equilibrium is reached. The resulting data, velocities, discharges, and nodal volumes for a complete cycle is read onto tape at intervals of 0.5 to 3 hours (real time) to serve as input to the companion quality models. Simulation of three cycles (about 3 days real time) is accomplished for a system of 200 nodes and 300 links with $\Delta t = 5$ minutes in about 4.5 minutes on a Univac 1108.

7-2.8 Dynamic Water-Quality Model

The Bay-Delta Dynamic Water-Quality Model[17,18,19,20] is formulated from the basic advection-effective diffusion equation in a manner similar to that described previously for the DECS Model. Differences lie in providing for branching and looping and in determining the concentration of a quality constituent in the links connecting node centers. The general finite difference equation is

$$\frac{d(VC)_j}{dt} = -\sum_{i=1}^{n} (QC)_i - \sum_{i=1}^{n} \left(EA\frac{dC}{dl}\right)_i + \Sigma s \tag{7-22}$$

where $(VC)_j$ = mass of quality constituent carried in node j

$\Sigma(QC)_i$ = algebraic sum of advective mass transport rates for link i connected to node j

Q_i = flow in link

C_i = concentration of quality constituent in link

$\Sigma[EA(dC/dl)]_i$ = algebraic sum of diffusional mass transport rates for links connected to node j

l_i = length of link

A_i = cross-sectional area of link i

E_i = effective diffusion coefficient for link i

s = source and/or sinks of mass in node j

Experience with the Bay-Delta System and other estuaries where the node-link network conceptualization has been applied has shown that when dynamic conditions are well represented and large numbers of discrete elements are used to describe the hydrodynamic behavior, Eq. (7-22) may be simplified by dropping the diffusional term. Thus, for any node j we have

$$\frac{d(VC)_j}{dt} = -\sum_{i=1}^{n} (QC)_i + \Sigma s \tag{7-23}$$

where n is the number of connecting links.

To illustrate the approach for formulation of the basic mass balance equation in finite difference form consider node 5 in the definition sketch of Fig. 7-5. Neglecting any sources or sinks other than those shown, the mass balance equation for node 5 becomes

$$\Delta(V_5C_5) = [\bar{Q}_{25}\bar{C}_{25} + \bar{Q}_{45}\bar{C}_{45} - \bar{Q}_{56}\bar{C}_{56}]\,\Delta t \tag{7-24}$$

in which $\bar{Q}$ and $\bar{C}$ indicate averages of these quantities over the finite time interval Δt.

Since concentrations and velocities are changing with time, and since advective mass transport only allows a particle to move a distance in time Δt proportional to the mean velocity, some difficulty is experienced in defining the concentrations brought into node 5 by $\bar{Q}_{25}$ and $\bar{Q}_{45}$. The simplest definition is to require that the C's be at the concentration of the node of origin, i.e., advective mass transport from nodes 4 to 5 would be $\bar{Q}_{45}\, C_4$ and from 5 to 6 it would be $\bar{Q}_{56}\, C_5$. Unfortunately, for an explicit solution technique this assumption results in numerical propagation of quality perturbations at rates greater than would be expected by advection alone. This defect has been corrected[20] by the practical device of interpolation of concentration between node centers and using C's corresponding to a point on the concentration profile a distance $\bar{u}\, \Delta t$ from the downstream node in the direction of flow.

A set of mass balance equations can be written for the entire network. An explicit step-forward solution technique yields temporal and spatial descriptions of quality for the estuary. Time steps of 0.5 to 3 hours are employed in the Bay-Delta Model with the condition that, for all links and time steps,

$$\Delta t \geq \frac{l_i}{\bar{u}_i} \tag{7-25}$$

where $\bar{u}_i$ is the average velocity corresponding to the average channel Q's in the finite difference equations. Violation of this condition invites instability.

Nonconservative constituents are also accommodated in the Dynamic Water-Quality Model by adding appropriate source or sink terms. Couples, or so-called feedback relationships such as the BOD-DO, nitrification, or the transfer of nutrients in ecologic succession, may be treated by adding equations consistent with the number of variables and unknowns. In the early versions of the Bay-Delta quality models up to 10 independent quality parameters of either conservative or nonconservative type plus the BOD-DO coupled relationship could be handled. Recently, this capability has been considerably expanded by Chen[22] in treating the role of nutrients in ecologic succession in aquatic environments. In one version of a modified Bay-Delta quality model 19 coupled mass balance equations were employed in a network of about 70 nodes and 130 links. A solution for this system involving simulation at 3-hour time steps for 8 days real time required about 3.5 minutes to execute on a Univac 1108.

Steady-State Water-Quality Model

The Bay-Delta version of the steady-state approach is based on the assumption that, for a given set of steady flows and loads,

$$-\sum_{i=1}^{n} (\bar{Q}\bar{C})_i + \sum_{i=1}^{n} \left(\bar{E}\bar{A}\frac{d\bar{C}}{dl}\right)_i + \Sigma s = 0 \tag{7-26}$$

i.e., there is no net change with time in mass of constituent $\bar{C}$ in node j due to the combined effect of advection, effective diffusion, and sources and sinks (decay, reaeration, growth, etc.). If this condition prevails, and the quantities $\bar{Q}$, $\bar{A}$, $\bar{E}$ and the coefficients of s are known and constant, then Eq. (7-26) written for each node in the system results in a set of n equations with n unknowns, the nodal concentrations. This set is sparse, with few coefficients, and unsymmetrical. It is most conveniently solved by an iterative technique given a reasonable starting condition. A modified Gauss-Seidel scheme is employed and convergence is rapid, usually to within about 1 percent in three or four cycles.

The steady-state model requires diffusion coefficients. These are derived most conveniently from a preliminary application and verification of the Dynamic Water-Quality Model against prototype behavior. Since steady-state conditions per se are seldom even closely approximated in the prototype, the model and the results derived from it must be judged accordingly. As a practical tool for evaluation of many alternatives in complex estuarial systems the model has proved very useful. Computer costs are very low: about 1.5 minutes of Univac 1108 time is required to achieve a solution for a system involving 2 sets (one unknown each) of 200 equations each.

7-2.10 Gulf Coast Models

Among the most versatile estuarial models currently in use are a set developed by Masch and his associates at the University of Texas. These are a two-dimensional hydrodynamic model for shallow, vertical mixed embayments, of which the Gulf Coast estuaries are typical, and companion quality prediction models designed to be used interactively with the hydrodynamic model in much the same manner as the Bay-Delta Models previously described.

7-2.11 Two-Dimensional Hydrodynamic Model—HYDTID

The two-dimensional Hydrodynamic Model was inspired in part by the work of Reid and Bodine,[26] who adapted the basic equations set forth by Dronkers[4] to a solution of the problem of long-period hurricane surges in Galveston Bay. Masch et al.[27] had independently pursued the tidal hydrodynamic problems of the shallow Gulf Coast estuaries, including Galveston Bay, and found that the scheme proposed by Reid and Bodine facilitated solution. Subsequently, the model, named HYDTID, was substantially improved by Masch and his associates through practical applications to Galveston Bay[1] and other shallow estuaries along the Texas coastline.[5,28] It is fully operational at present and has been more widely applied to practical studies of real estuarial systems than any other two-dimensional hydrodynamic model known to exist in the United States today.

HYDTID employs an orthogonal pattern of "cells" to approximate the estuarial configuration and to define certain necessary physical and mathemati-

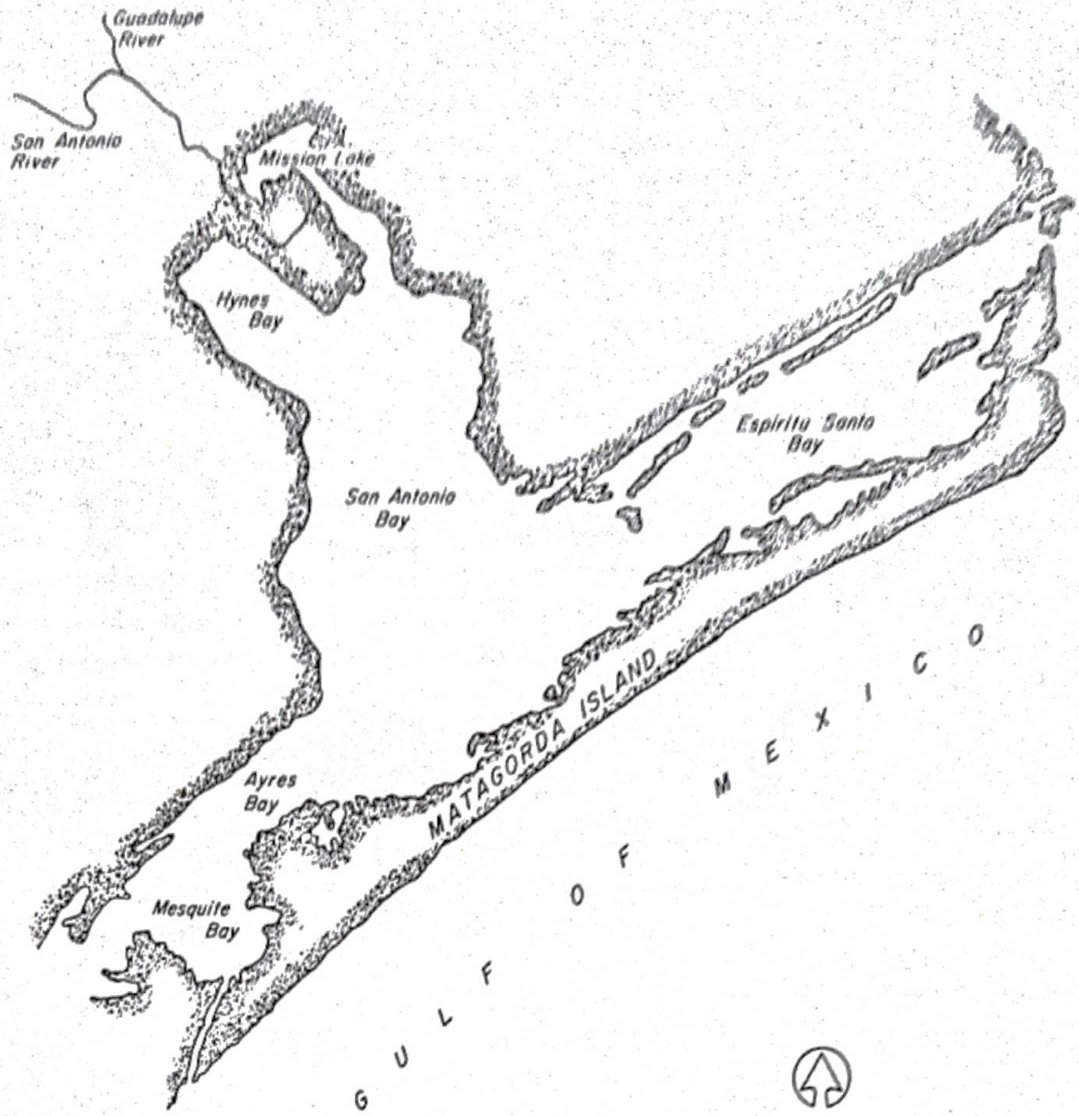

FIGURE 7-6
San Antonio Bay, Texas.

cal conditions. The conceptual adaptation of the model is illustrated by the example of San Antonio Bay in Figs. 7-6 and 7-7 and by the definition sketch of Fig. 7-8.

Application of the model entails superposition of a computational grid of dimensions appropriate to the detail desired by the modeler and governed to some degree by the geometric configuration of the prototype. In practice, grids involving as many as 1,000 cells have been employed.

Cells are treated as both flow and volume elements, as contrasted to the Bay-Delta approach, and for practical reasons are of square form in plan. To facilitate the solution and to allow flexibility in adapting the model to unusual boundary and flow conditions, a wide variety of cell types are identified

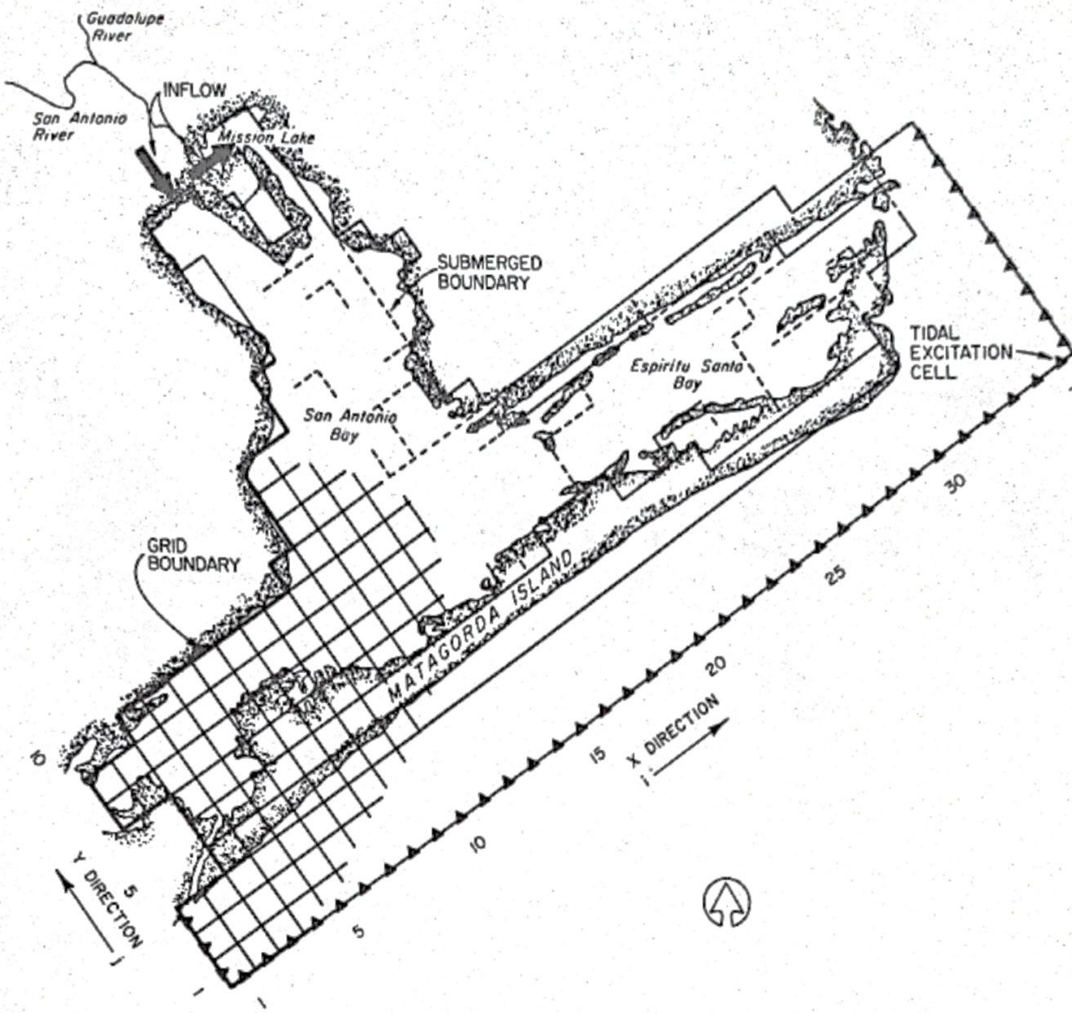

FIGURE 7-7
Computational grid adapted to San Antonio Bay, Texas.[28]

("flagged"). These are categorized as to general type as:

1 *External boundary.* Cells outside the perimeter of the boundary being modeled but adjacent to the boundary, e.g., inflow, no flow, outflow, or ocean tidal cells.

2 *Internal boundary.* Cells defining internal discharges, exports, barriers (no flow or restricted flow), tidal conditions at ocean boundaries, and landward boundaries.

3 *Internal water.* Cells within the system with no boundary or barrier restrictions.

4 *Dead.* Cells not used in computation but needed to preserve continuity in the indexing system.

Examples of a few of these cell types are illustrated in the sketch of Fig. 7-8. Some 30 different cell flag designations are employed in HYDTID. Variable

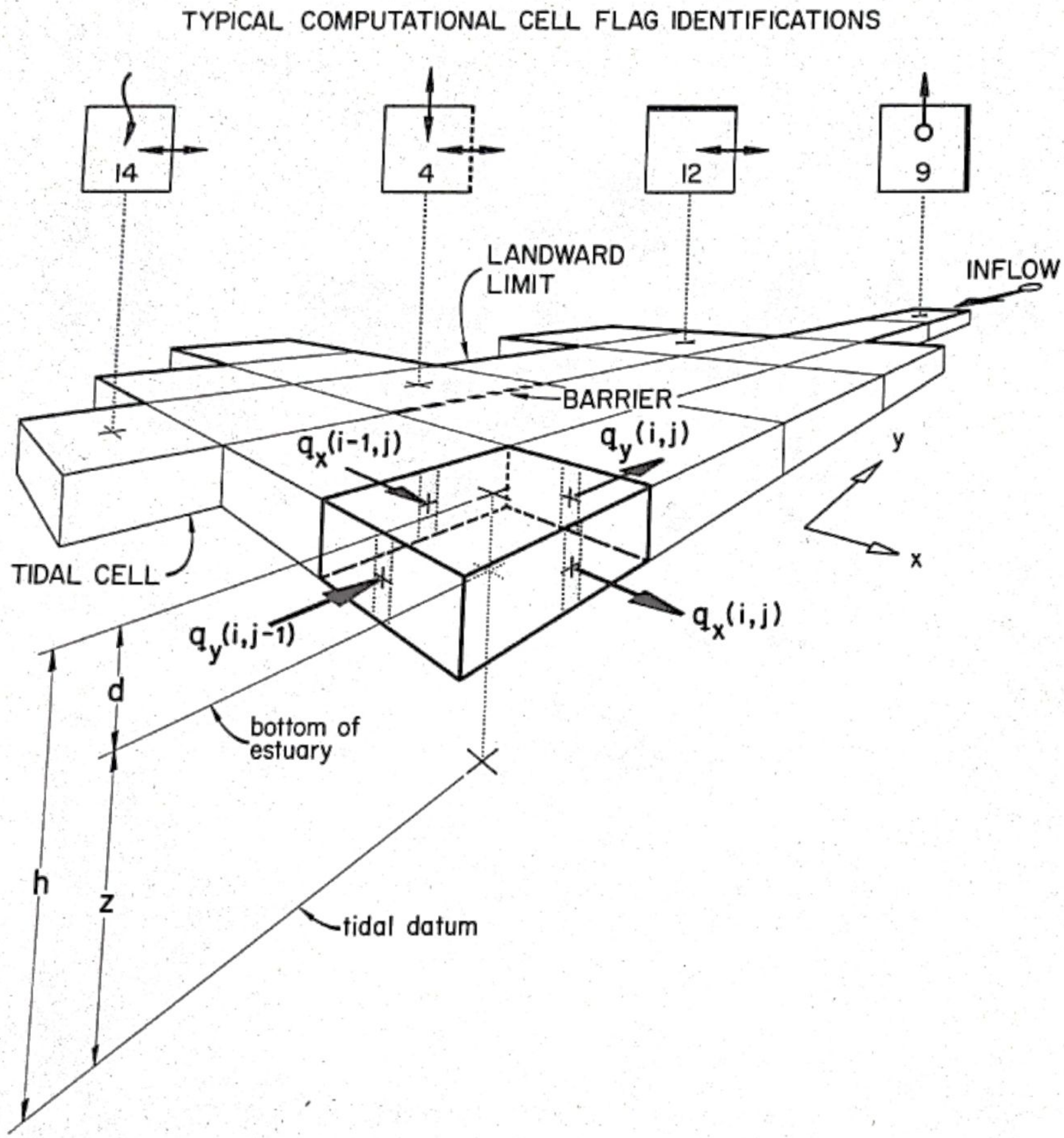

FIGURE 7-8
Definition sketch—Masch two-dimensional orthogonal grid hydrodynamic model.[5]

definitions are as previously given, all except tidal flows being identified with the center of a cell as illustrated in Fig. 7-8. For indexing purposes the indices i and j are taken as increasing along the x and y axes, respectively.

Formulation of the equations of motion and continuity is identical to that set forth earlier in the section on theoretical considerations [Eqs. (7-5), (7-6), and (7-7)]. For numerical solution, an explicit technique with a time-centered difference scheme of the so-called leapfrog type is employed. Time notations for the recursive relations are

$$t - 1 = (K - \tfrac{1}{2})\,\Delta t$$

$$t = K\ \Delta t$$
$$t + 1 = (K + \tfrac{1}{2})\ \Delta t$$

and
$$t + 2 = (K + 1)\ \Delta t$$

where k is an integer.

The three basic unknowns, the tidal flows q_x and q_y and the tidal (water surface) elevation h at time $t + 1$, are solved by the difference equations:

For tidal flows:

$$q_x^{t+1}(i,j) = \frac{1}{C_x^{t-1}}\left\{q_x^{t-1}(i,j) + g\ \Delta t \left[\frac{d^t(i,j) + d^t(i+1,j)}{2}\right] \times \left[\frac{h^t(i,j)\, h^t(i+1,j)}{\Delta x}\right] + X_w^t(i,j)\ \Delta t + \Omega \bar{q}_y^{t-1}(i,j)\ \Delta t\right\} \tag{7-27}$$

and

$$q_y^{t+1}(i,j) = \frac{1}{C_y^{t-1}}\left\{q_y^{t-1}(i,j) + g\ \Delta t \left[\frac{d^t\,3(i,j) + d^t(i,j+1)}{2}\right] \times \left[\frac{h^t(i,j) - h^t(i,j+1)}{\Delta y}\right] + Y_w^t(i,j)\ \Delta t - \Omega \bar{q}_x^{t-1}(i,j)\ \Delta t\right\} \tag{7-28}$$

For tidal elevation:

$$h^{t+2}(i,j) = h^t(i,j) + \Delta t\left[\frac{q_x^{t+1}(i-1,j) - q_x^{t+1}(i,j)}{\Delta x} + \frac{q_y^{t+1}(i,j-1) - q_y^{t+1}(i,j)}{\Delta y} + r^{t+1}(i,j) - e^{t+1}(i,j)\right] \tag{7-29}$$

The values $\bar{q}_x$ and $\bar{q}_y$ are the averages of the four tidal flows corresponding to the indices (i,j), $(i,j+1)$, $(i-1,j+1)$, and $(i-1,j)$. The coefficients C_x and C_y incorporate the effects of frictional forces and are computed from tidal flows at time $t - 1$ and depths at time t.

In order to maintain a stable solution it is necessary to observe the criterion that

$$\Delta t \leq \frac{\Delta S}{\sqrt{2\ g d_m}} \tag{7-30}$$

where ΔS is the cell size and d_m is the maximum depth in the estuarial system.

Results derived from a simulation are temporal histories of component tidal velocities and water surface elevations for each cell in the grid. Special routines have been designed to produce mean velocities, mean depths, resultant velocities, and graphical plots of current circulations and particle travel. Average depths and average component velocities are read onto tape for use in the companion water quality models.

7-2.12 Salinity Prediction Model

Brandes and Masch[29] envision the development, ultimately, of three mass transport models that will be interactive with HYDTID: a short-term, a long-term, and a steady-state model. Each of these models is based on the advection-effective diffusion equation in the general form of Eq. (7-4). All have been developed to the operational stage for conservative substances. Model development has been well documented in a series of technical reports[5,28,29,30] and is much too detailed to present here. It may suffice to briefly describe the solution technique employed in developing the dynamic quality model for salinity prediction in what Masch et al.[29] term the "SAL System."

In this model each of six terms in the two-dimensional diffusion equation is written in finite difference form for solution by the Alternating Direction Implicit (ADI) technique.[31] The variables are defined in a space-staggered scheme compatible with that employed in the hydrodynamic model. Salinities (or other conservative substances) are defined at the center of each computational cell, and the net convective velocity components $u(i,j)$ and $v(i,j)$ are defined at the right and upper sides of each cell, respectively.

Two sets of finite difference equations are written: one for time level $(t + 1)$ involving the x derivatives in implicit form and the y derivatives in explicit form, and the second for time level $(t + 2)$ involving the y derivatives in implicit form and the x derivatives in explicit form. Two sets of simultaneous equations in C result that can be solved directly without iteration.

Results of application of the SAL System are temporal and spatial variations of conservative substances. Experience to date has been limited to estuaries along the Gulf Coast, including San Antonio Bay and Matagordo Bay,[20] and to one in North Carolina, but the model shows promise of utility in studies of water management involving shallow estuaries subject to salinity intrusion. It is planned to extend the model's capability by including other quality constituents.

7-2.13 Rand Model

J. J. Leendertse of the Rand Corporation has developed a mathematical model package capable of both hydrodynamic and water-quality simulation for "well-mixed estuaries and coastal seas." It is composed of a two-dimensional hydrodynamic model that solves the basic differential equations of motion and continuity by a time-centered difference scheme and an integrated quality simulation routine based on the mass balance equation that solves the quality equations by an alternating direction implicit-explicit technique.

7-2.14 Two-Dimensional Hydrodynamic Model

The Leendertse-Rand hydrodynamic model[32] is somewhat more rigorous than some of those previously described, in that a particular effort is made, at least at the outset, to preserve all terms in the fundamental differential equations. For example, the Bernoulli acceleration (convective) terms

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

and

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}$$

are retained, a requirement not considered necessary by Shubinski[23] and Masch et al.[27] in their studies of shallow, vertically mixed estuaries. It is presumed that these terms may be dropped in those cases where they are of little importance, with corresponding savings in computer time, although this does not seem to have been of major concern in Leendertse's developmental work.

Leendertse emphasizes the importance of error propagation and amplification resulting from lack of rigor in solution of the equations and in correctly describing boundary conditions for the models. He has been particularly cautious to "center" his finite difference equations and to assure consistency and accuracy in results.

Conceptually, the Rand Model package employs a two-dimensional grid, similar in certain respects to that used by Masch and his associates. The grid size may be varied to suit the detail required in space. However, Leendertse avoids some problems of solution stability by employing a unique set of approximation equations for the basic motion and continuity relations. These result in a nearly centered solution scheme in which the primary consideration in selection of Δt is accuracy. The system of equations with unknowns in u, v, and h is solved by recursion formulas in a stepping procedure that alternates from implicit to explicit and is inherently stable. The procedure is described in considerable detail by Leendertse.[32,33,34,35]

Use of the model in the United States appears to have been limited to a special study of Jamaica Bay[35] and recently in a study of Tampa Bay, although use has been extensive in studies of parts of the North Sea and the estuary of the Rhine River.[36] Accuracy of the model, as exemplified by verification against tidal elevations, is considered excellent.

7-2.15 Water-Quality Model

Leendertse's water-quality model is similar in many respects to that of Brandes and Masch[29] described earlier. It is formulated on a grid system identical to that used for the hydrodynamic model and employs an alternating direction implicit-explicit technique for solution of a set of quality equations written in a staggered scheme over the grid space. A unique difference lies in the incorporation of the quality routines directly into the estuarial model package; in essence, the quality model is really an extension of the earlier hydrodynamic model to include a new dimension, quality. It should be noted that, in making this extension and applying it to a study of Jamaica Bay, Leendertse dropped the "advective" term $u\ \partial u/\partial x$ from the computation, apparently with the tacit acknowledgment that, for this case, at least, such terms are negligible.

The quality model is capable of describing the fate of both conservative and nonconservative quality constituents in shallow, vertically mixed estuaries or "coastal seas." Up to six quality constituents, either conservative or non-conservative first-order, can be handled simultaneously. Coupled relationships such as the DO-BOD interaction are provided for in the model. Graphical plotting routines have been provided to assist the analyst in interpretation of the results of simulations and in determining the sensitivities of the model to data and assumptions.

7-2.16 Triangular-Element Hydrodynamic Model

Shubinski[23] adopted a system of triangular elements to describe the two-dimensional flow regime in a shallow portion of South San Francisco Bay. A typical triangular element, which can be of any shape and size, and suited to the geometric and hydrodynamic requirements of the problem being studied, is illustrated in Fig. 7-9. The element is characterized by a mean depth, surface area, volume, and bottom roughness (either isotropic or anisotropic), and is oriented in relation to fixed coordinate axes by stipulation of the coordinates of the vertices.

Each vertex defines a node center to which is allocated one-third of the volumes of each of the surrounding elements, defined by the medians of the triangle and the adjacent sides. The resulting node is polygonal, with two sides defined by each contiguous element. It has characteristics of mean depth, surface elevation, surface area, and volume.

The two-dimensional equations of motion (eliminating the Bernoulli accelerations) are written for an element as

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} - g\,S_{ex} + \frac{X_w}{d} - \frac{\Omega v}{d} \tag{7-31}$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial y} - g\,S_{ey} + \frac{Y_w}{d} + \frac{\Omega u}{d} \tag{7-32}$$

where u, v = mean velocities in the x and y directions
d = mean depth in the element

and other terms are as previously defined. Continuity is written for each nodal volume as

$$\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0 \tag{7-33}$$

where q_x, q_y are unit flows in the x and y directions.

Solution of the motion and continuity equations is accomplished by a two-step explicit technique and results in time histories of velocities within each ele-

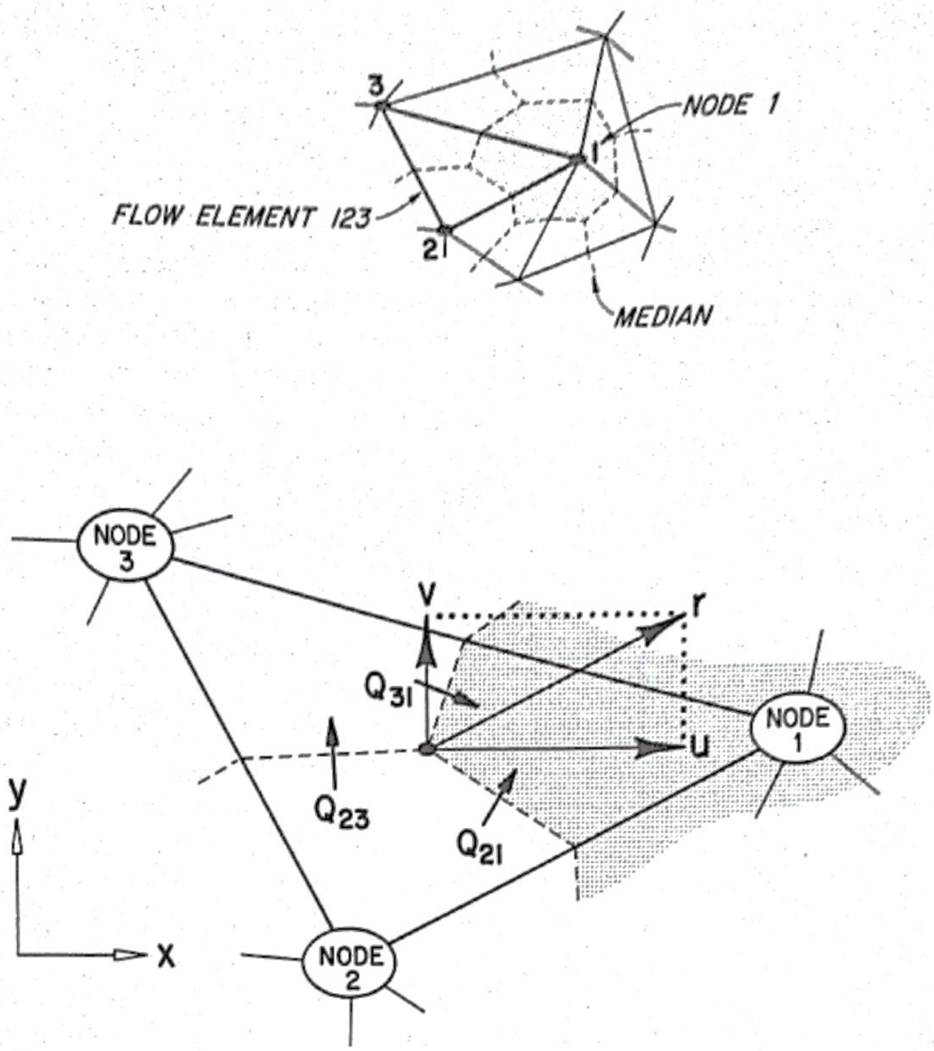

FIGURE 7-9
Definition sketch—WRE two-dimensional triangular-element hydrodynamic model.

ment. Flows are transferred between nodes, i.e., across median lines within the elements, in accordance with the velocity field within the elements.

Applications of this technique have been in the investigation of proposed bridge crossings of South San Francisco Bay, where scour and deposition of sediments may be problems. However, the method is general and should be suitable for study of other types of problems. Advantages are associated primarily with the fact that triangular elements can be used; elements can be varied in size, shape, and orientation to give the modeler the maximum opportunity to correctly describe the flow around obstacles in the estuary or in areas of special interest from the quality viewpoint. In one of the cases studied, a system of 132 elements and 85 nodes was employed. Elements vary in size from about 1,500 feet on a side to nearly 25,000 feet. A stable dynamic solution* for a complete tidal cycle was obtained by simulation of three cycles after initialization. Solution time with $\Delta t = 1$ minute (real time) was about 15 minutes on a Univac 1108.

7-2.17 Estuary Ecologic Model

Chen[24] proposed a mathematical model of the aquatic ecosystem based on a set of mass balance equations of the general type employed in earlier water-quality models but involving some additional concepts derived from biokinetic principles. This model has been incorporated into the basic structure of the Bay-Delta Dynamic Water-Quality Model to form a new package, the Estuary Ecologic Model.

The quality and ecologic interactions envisioned by Chen for a succession of trophic levels from bacteria and phytoplankton, through zooplankton, to fish, and involving benthic animals are illustrated schematically in Fig. 7-10. The model considers two general kinds of mass balances for the integrated aquatic environment, one for abiotic substances and one for biota.

In all, 23 mass balance equations are formulated for the model. Variables include: nutrients (CO_2, NH_3—N, NO_2—N, NO_3—N, PO_4), BOD, DO, temperature, light energy, pH, alkalinity, chlorides, TDS, algae (three groups), zooplankton, fish (three groups), benthic biomass, detritus, and certain physical properties of the system. Where substances move with the liquid phase, they are subjected to the advection-effective diffusion mechanisms usually adopted for quality models, and consequently the appropriate coefficients must be provided.

The concepts embodied in the model and details of its development are documented elsewhere,[22,24,25] but it may be instructive to illustrate the approach by writing a few of the equations for biological interactions:

For a nutrient:

$$\frac{d(\bar{V}C)}{dt} = [A + D] - \frac{1}{Y_p} \sum_{i=1}^{n} \mu_i \, p_i + \alpha Z + \beta D \qquad (7\text{-}34)$$

For phytoplankton (single group):

$$\frac{d(\bar{V}p)}{d} = [A + D] + (\mu - r - s)p - \frac{1}{Y_z} gpZ \qquad (7\text{-}35)$$

* The criterion for assuring a stable solution is that, for all elements,

$$\Delta t \leq \frac{l}{c}$$

where $c \cong \sqrt{gd}$

l = length of the shortest side of the triangular element

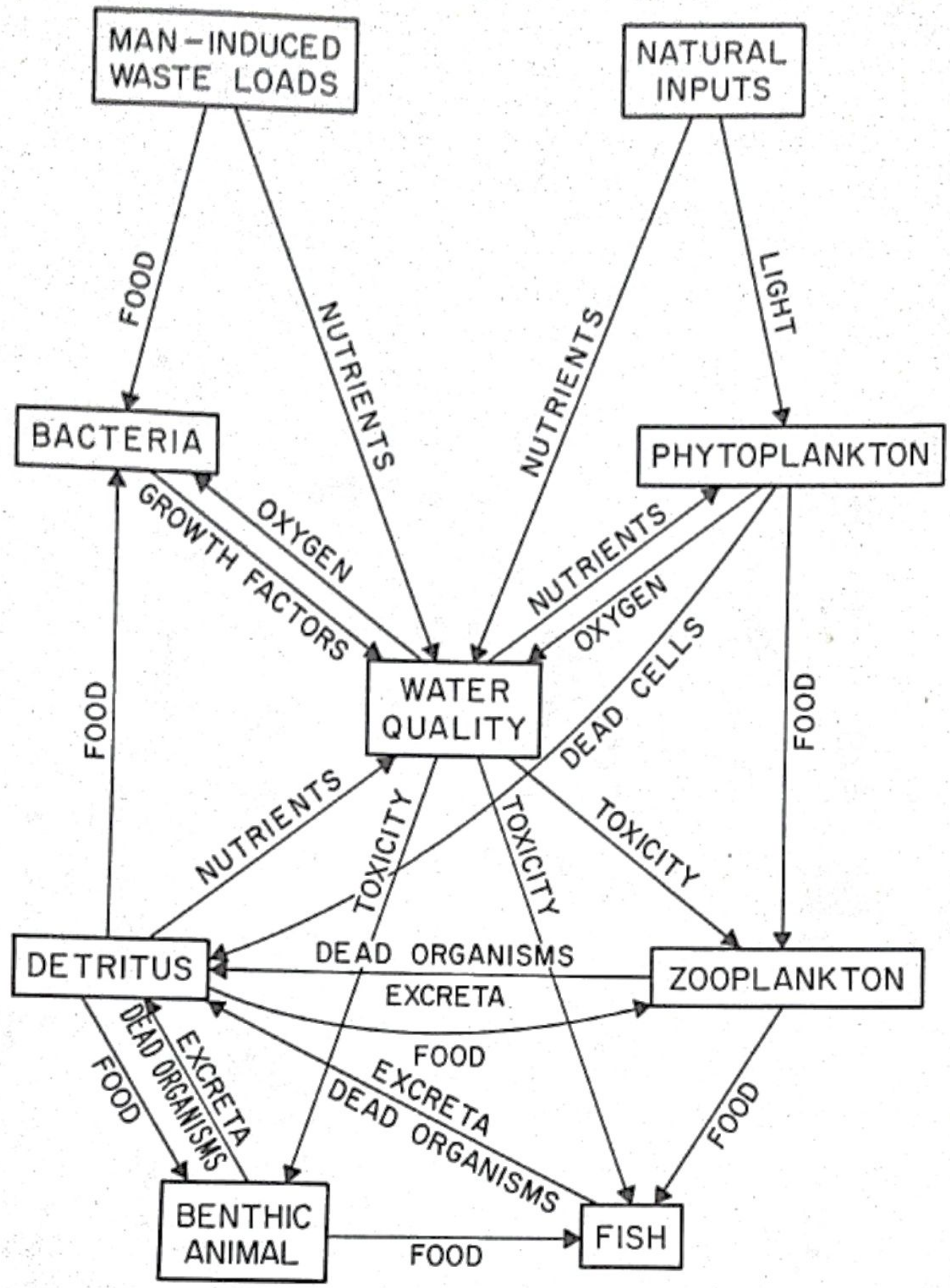

FIGURE 7-10
Definition of an aquatic ecosystem.[22]

For zooplankton:

$$\frac{d(\bar{V}Z)}{dt} = [A + D] + Z \sum_{i=1}^{n} g_i \, p_i - mZ - \frac{1}{Y_f} \gamma F \qquad (7\text{-}36)$$

where $[A + D]$ = advection and effective diffusion of indicated substance
C = concentration of nutrient

p = biomass concentration of phytoplankton (algae)
Z = biomass concentration of zooplankton
F = biomass concentration of fish
D = benthic biomass concentration
μ = growth coefficient for algae governed by Michaelis-Menton limiting growth factor relationship
r = respiration coefficient for algae
s = algal cell sedimentation rate
α = zooplankton nutrient utilization rate
β = benthic biomass nutrient utilization rate
g, γ = grazing coefficients
Y_p, Y_z, Y_f = digestion efficiencies, yield coefficients
i = algal group
n = number of algal groups

Chen has designed the ecologic model for adaptation to a variety of aquatic environments, including shallow, vertically mixed estuaries. His scheme for estuaries is based presently on the Bay-Delta node-link configuration and has been applied preliminarily to this system.[22] While data are meager for verification of certain predictive capabilities provided for in the model, especially in the biological aspects, the model gives a good accounting of many of the parameters traditionally used to describe quality changes, e.g., salinity, BOD, DO, temperature, pH, alkalinity, etc. Estimates of biological response are in general agreement with field experience but require supporting data currently being acquired. The models are currently being used to give guidance in evaluation of alternative water-quality control schemes, the most valuable function of any water-quality model.

7-3 MODEL TESTING

The test of the model's capability usually is found in how well it represents the behavior of the prototype for some known historic record. This process has been loosely called *verification*. Actually, the term is a misnomer, since by definition a model is not the prototype, but a rather crude representation that cannot be expected to perform exactly as we observe the real estuary to behave in nature.

Nevertheless, it is necessary to confirm the model's conceptual and functional approximation of the real system by carrying out some sort of comparison of simulated and observed performance. Adjusting the model to improve agreement with prototype experience is called *calibration*. The capability of the calibrated model to reproduce prototype estuarial hydrodynamic behavior has usually been assessed in terms of tidal elevations at points that lie inside the model domain. This seems to have been accomplished in a satisfactory way for each type of hydrodynamic model discussed here, as evidenced by the comparisons shown in Fig. 7-11. Confirmation of current velocities and tidal flows is

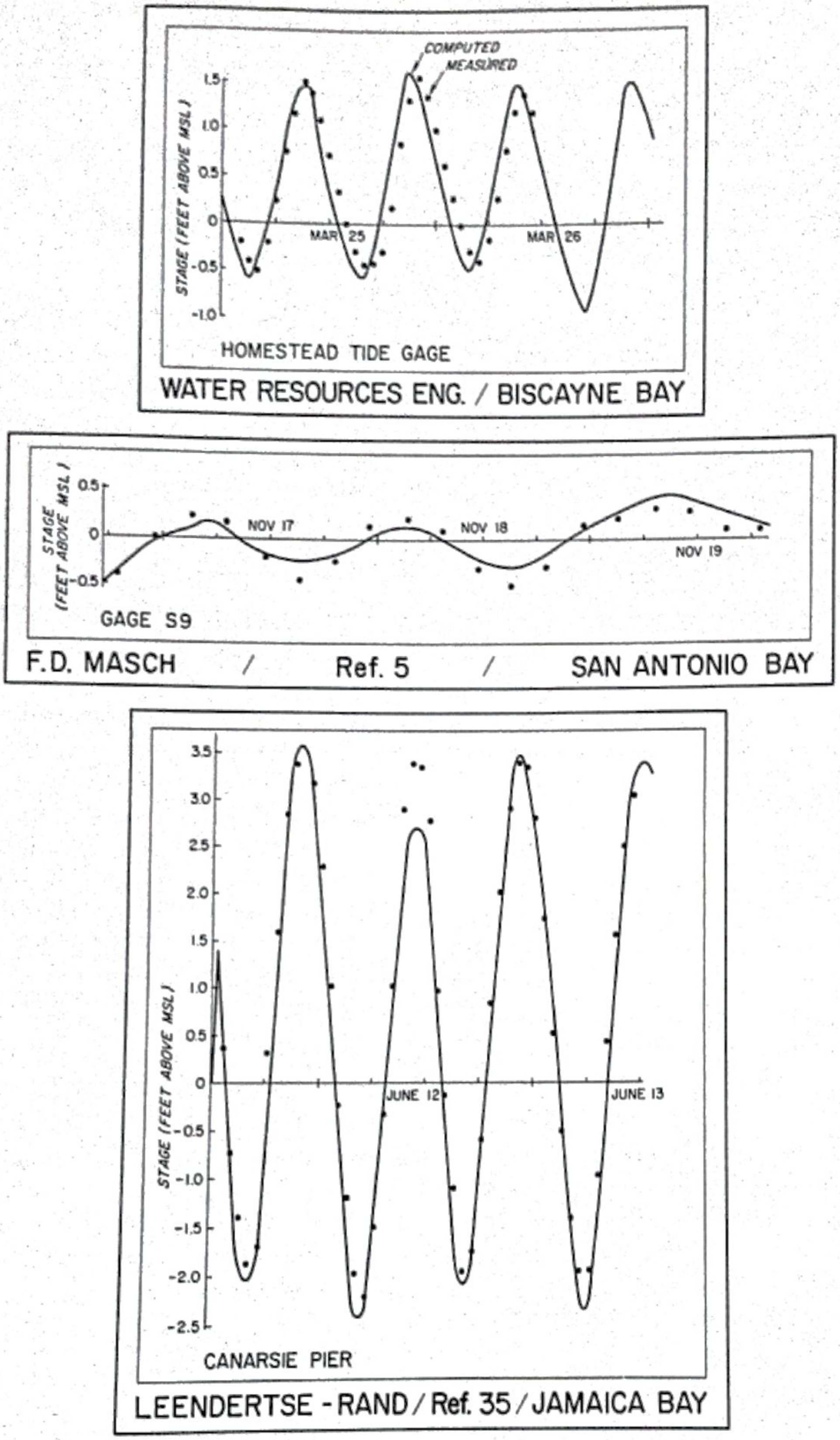

FIGURE 7-11
Verification of tidal amplitude, estuarial hydrodynamic models, selected examples.

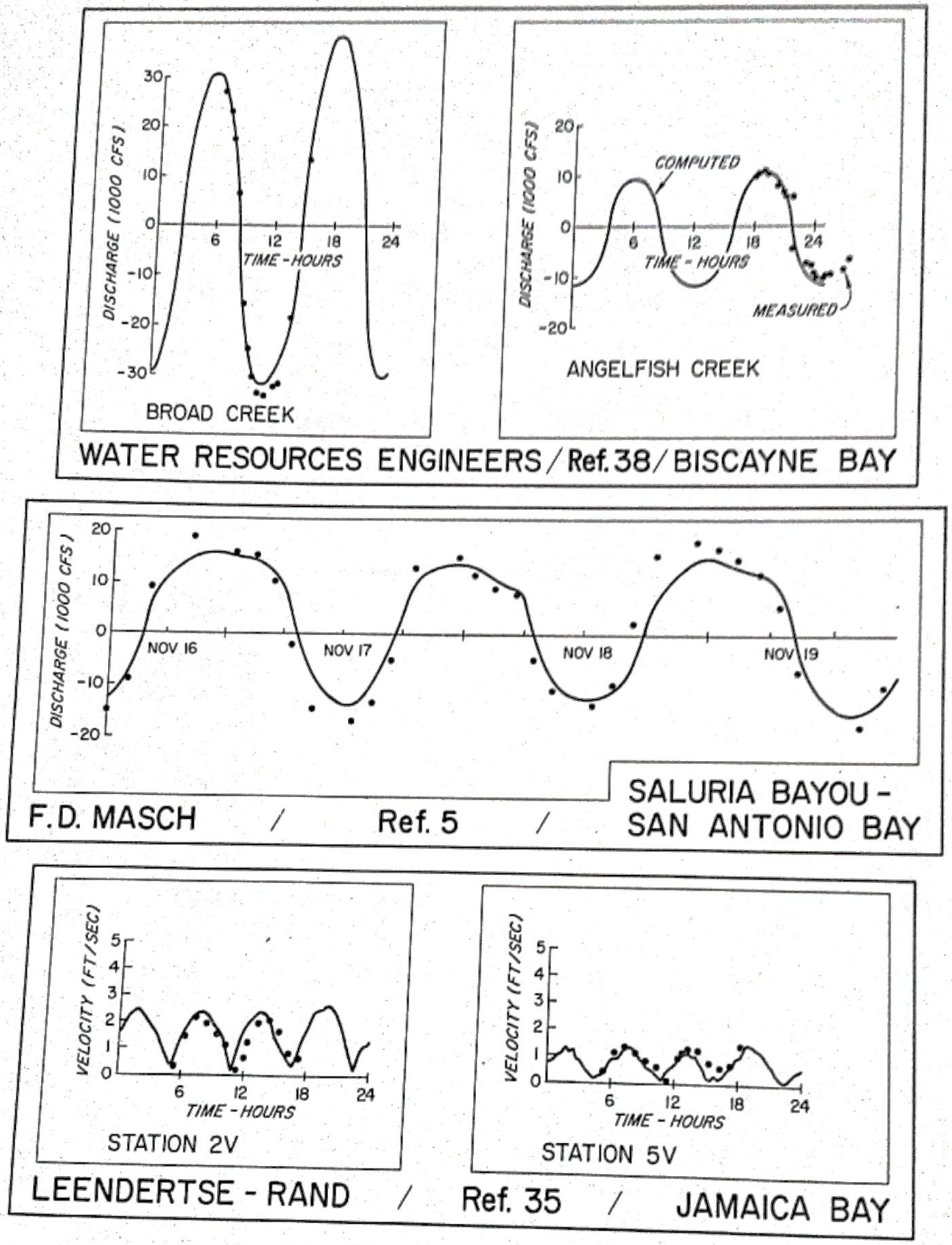

FIGURE 7-12
Verification of tidal flows and velocities, estuarial hydrodynamic models, selected examples.

somewhat more difficult due to the complications of field measurement. Good data on velocities and discharges within estuarial systems are scarce, but when such data can be secured, as in the examples illustrated in Fig. 7-12, it is possible to secure reasonable agreement between model prediction and field measurement.

Success in testing water-quality models has generally been less than satisfactory, largely because of the paucity of good data but also because of the greater dependence on empiricism in structuring the models. As noted previously, the relative dependence on the advection and diffusion terms in the mass transport equation is a function of the scale of the model. As the scale increases, i.e., as segments become larger or phenomena are averaged over longer time steps, the dependence on the empirical effective diffusion coefficient increases. Calibration of coarse-scale mathematical models is usually accomplished by subjective adjustment of the coefficient until the model prediction agrees with prototype performance. Because the coefficients are "derived" from historic experience with the prototype, they are not usually considered reliable for prediction of prototype performance under conditions that differ markedly from the historic.

The greatest success in quality simulation has usually been obtained for mass transport of conservative substances, e.g., salinity. The Bay-Delta Models have been tested extensively[1,14,21] in their capability to represent the salinity intrusion-repulsion phenomena, and the models developed by Masch et al.[27] have been well checked in their capability to describe the salt balances of several Gulf estuaries. The Leendertse-Rand Model has been successfully checked on Jamaica Bay.[35]

Success in substantiating the DO-BOD relationship with deterministic estuary models has not been particularly notable. Perhaps, the best results have been obtained with the DECS Model on the Delaware estuary,[2,6,7] where the model gave a fair representation of the seasonal average oxygen sag along the estuary. Also, the Bay-Delta Models have given a fairly good accounting of seasonal oxygen deficiencies in the heavily polluted South Bay. Capabilities to represent short-term temporal variations in DO, such as provided for in the Chen Estuary-Ecologic Model[22,25] have not yet been verified for estuarial systems by direct comparisons. Spot field observations have been employed to corroborate the model's ability to correctly simulate diurnal variations in DO, but data are not available for detailed comparisons.

Much still needs to be done to formalize the techniques for assessment of model reliability. Some statistical measures of agreement between model and prototype are needed. A systematic procedure for exploring the sensitivities of models to data input, coefficients, boundary conditions, computational procedures, and other modeling assumptions would be a valuable addition to the modeling "art."

7-4 MODEL APPLICATIONS

Practical applications of estuarial mathematical models are not well documented in the technical literature; one must refer to project reports of special study programs to obtain a perspective on the real utility of such tools. Table 7-1 presents a brief summary of those cases known to the author in which the models described earlier in this chapter have been applied. The list is not comprehensive, but it is considered representative of the present state of the practitioner's art in the United States. Citations in the table are keyed to the list of references to assist the reader if he wishes to inquire further into the details of the applications.

To illustrate some recent uses of the models previously described and to point the direction of future development, three cases have been chosen for brief description here:

1 Applications of the HYDTID and SAL systems to San Antonio Bay, Texas, in a study of salinity balance (Fig. 7-13)

2 Application of the WRE models to study of potential scour-deposition problems resulting from proposed land filling of San Francisco Bay (Fig. 7-14)

3 Preliminary application of Chen's Estuary-Ecologic Model to San Francisco Bay and Delta (Fig. 7-15)

7-4.1 San Antonio Bay Salinity Balance

The models created by Masch et al. have been applied to San Antonio Bay in a program for the Texas Water Development Board[28] aimed at development of a methodology for predicting the ecological behavior of the estuarial system. Important elements of the program include development of a predictive capability for hydrodynamic behavior of the shallow estuary and characterization of the system's salt balance, which is influenced by upstream flow regulation and which in turn affects the ecologic response of the estuary.

The initial phase of the program involved application of HYDTID to predict the tidal flow regime in the estuary. Results exemplifying this application are present in Figs. 7-11 and 7-12, dealing with model verification. Subsequently, hydrodynamic data, tidal flows, and water depths were supplied to SAL, a model devised to solve the salinity transport problem. Results of the quality simulation are presented for two extremes of salinity intrusion and repulsion in Fig. 7-13.

Development under this program is continuing under the auspices of the Texas Water Development Board. It is planned to extend the model capability to include prediction of ecologic response, i.e., growth and death characteristics of indigenous aquatic biota, using the ecologic model concepts developed by Chen.[22]

Table 7-1 SELECTED EXAMPLES OF MATHEMATICAL APPLICATIONS TO ESTUARIAL SYSTEMS

Estuary (bay)	Location	Model description	Applications	Reference
Delaware	Del., Pa., N.J.	DECS, 1D SS Quality—30 segm.	Waste loading, DO	1, 6, 7, 8
Potomac	Md., Virginia	DECS, 1D SS Quality—28 segm.	Waste loading, DO, salinity	1
Hillsborough Bay	Florida	DECS, 2D SS Quality—35 segm.	Salinity balance	1
San Francisco Bay-Delta	California	(*a*) Bay-Delta (Hydr. Dyn. Q, SSQ) (200 nodes, 300 links)	Waste loads, salinity balance (Cl^-, TDS), water-level control, export pumping, wind effects, outfall location, land filling, scour, deposition	17, 18, 19, 20
		(*b*) WRE 2D Hydr. Δ element (85 nodes 132 elements)	Effects of bridge crossing, scour, deposition	23
		(*c*) WRE Ecologic (Hydr. + Dyn .WQ) (70 nodes, 130 links)	Ecologic response (DO temp, nutrients, biomass, pH, alk, etc.,	25
Humboldt Bay	California	Bay-Delta (Hydr. Dyn. WQ) (52 nodes, 113 links)	Waste loading, coliforms	—
San Diego Bay	California	FWQA-WRE (Hydr. Dyn. WQ) (dimension unknown)	Waste loading, coliforms	1
Columbia River	Ore.-Wash.	FWQA-WRE (Hydr. Dyn. WQ) (dimensions unknown)	Salinity balance	1
Sydney Harbor	Australia	FWQA-WRE (Hydr. Dyn. WQ) (173 nodes, 188 links)	Salinity, storm overflows, coliforms	17
Port Phillip	Melbourne, Aust.	Bay-Delta (Hydr. Dyn. WQ) (77 nodes, 190 links)	Salinity, wasteloads	17
Biscayne Bay	Florida	WRE (Hydr. Temp.) (97 nodes, 220 links)	Thermal balance	38
Galveston Bay	Texas	Masch-Tracor (Hydr. WQ Temp) (30×40 grid)	Salinity, thermal balance	1
San Antonio Bay	Texas	Masch et al. (HYDTID, SAL) (36×24 grid)	Salinity balance	28
Matagorda Bay	Texas	Masch et al. (HYDTID, SAL) (33×32 grid)	Salinity balance	28
Jamaica Bay	New York	Leendertse-Rand (Hydr.-WQ)	Salinity, waste loads, DO, coliforms	35

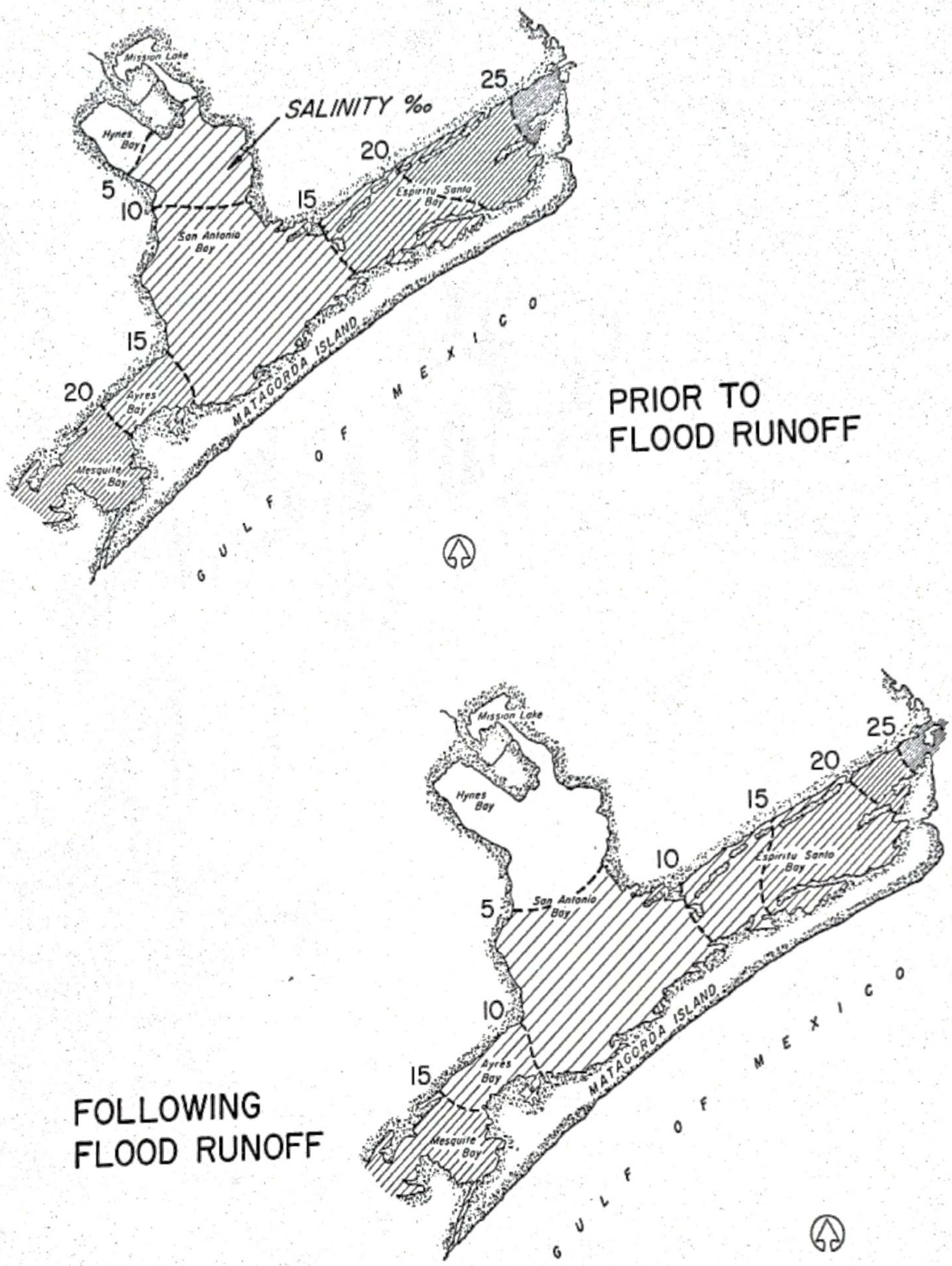

FIGURE 7-13
Salinity intrusion and repulsion, San Antonio Bay, Texas.[28]

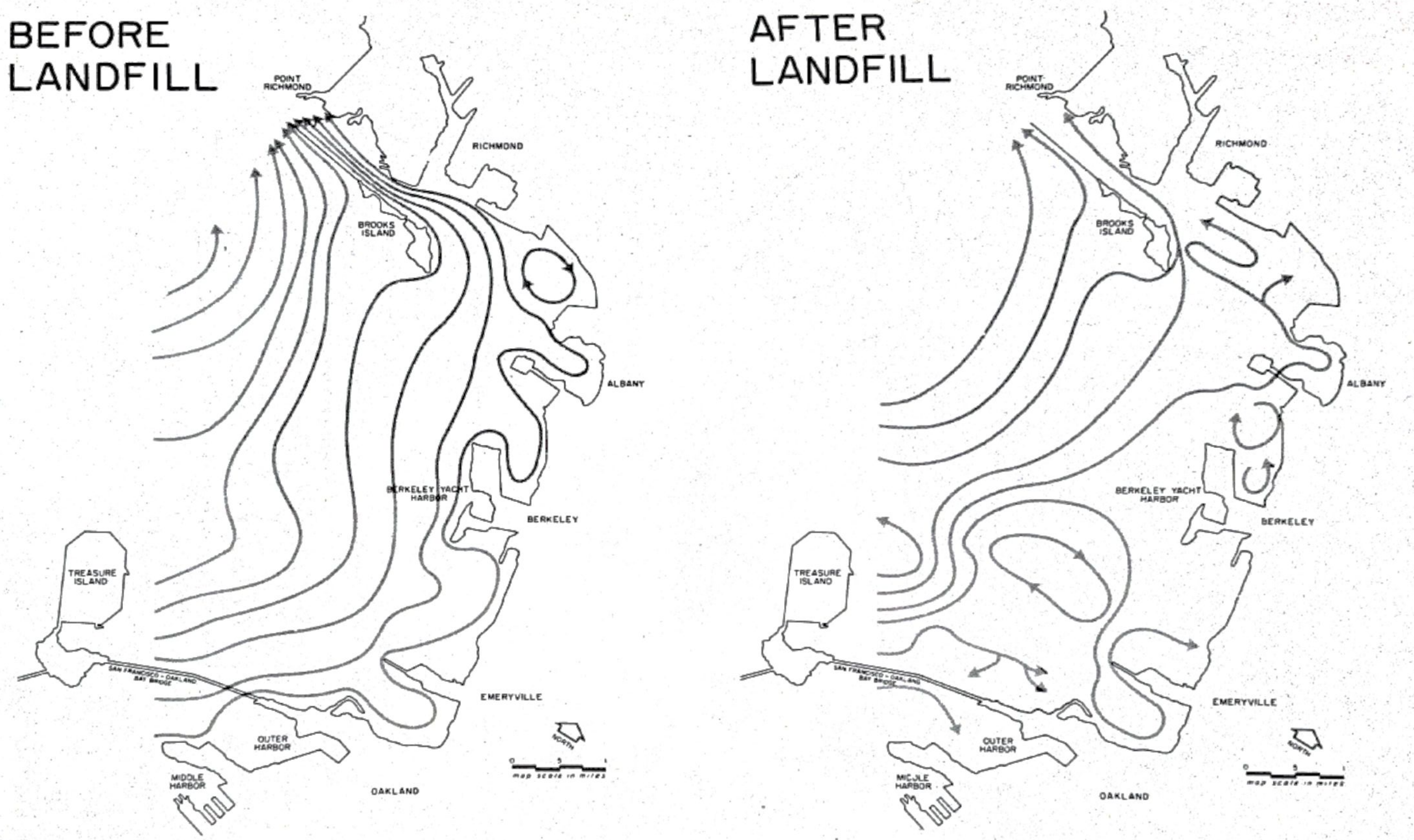

FIGURE 7-14
Effect of landfills on current directions and velocities, San Francisco Bay.[37]

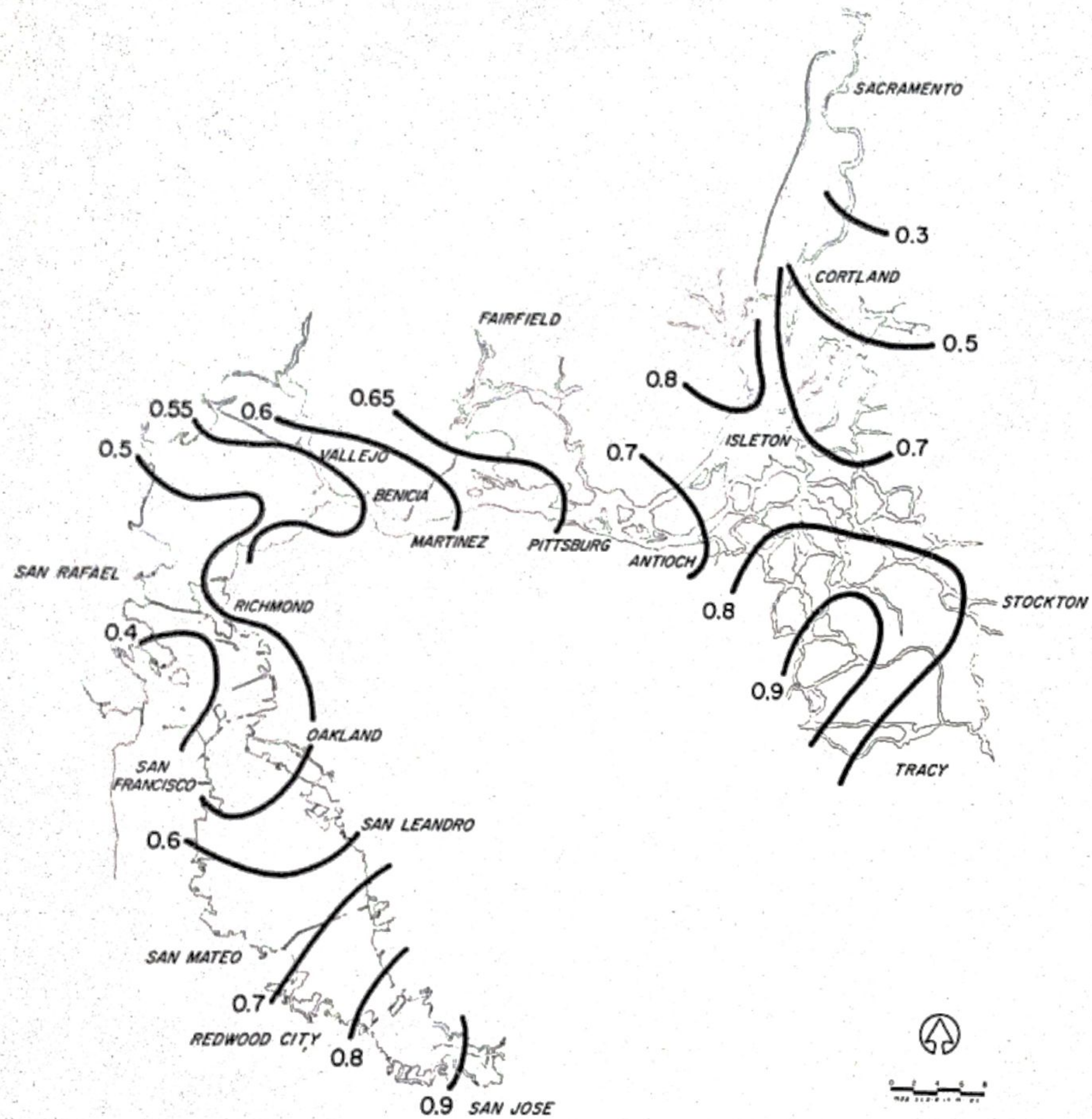

FIGURE 7-15
Distribution of algal biomass in Bay-Delta System, case study.[25]

7-4.2 San Francisco Bay Fill Investigation

The WRE versions of the Bay-Delta Models have been applied to a special study of the consequences of proposed land-filling operations on the hydrodynamic and water-quality behavior of San Francisco Bay.[37] Among the factors investigated are changes induced by filling on magnitude and direction of tidal velocities, tidal elevations, flushing of stagnant areas, dissolved oxygen resources, and scour and deposition of sediments.

Figure 7-4 shows the model configuration used in the study. The enlarged area illustrates the configuration of the submodel employed to explore the details of the system response in the region of proposed filling. Figure 7-14 presents some "before" and "after" results derived from model runs.

7-4.3 Estuarial Ecologic Model

The WRE Ecologic Model developed by Chen[25] has been applied in a preliminary assessment of alternative regional waste-water management schemes for San Francisco Bay. This assessment was part of a testing program undertaken under the sponsorship of the Office of Water Resources Research.[22]

The model has capabilities for representing the temporal and spatial distributions of aquatic biomass generated by nutrient loadings to shallow, vertically mixed estuaries. These may be presented as yields of phytoplankton, zooplankton, and fish per unit of estuarial area or volume or as concentration changes over any specified horizon from the diurnal cycle to the annual cycle.

Figure 7-15 illustrates graphically a typical pattern of algal productivity (shown as isopleths of algal biomass in milligrams per liter) over the Bay-Delta system (see Fig. 7-3).

REFERENCES

1. WARD, G. H., JR., and W. H. ESPEY, JR. (eds.), "Estuarine Modeling: An Assessment," Report to the Environmental Protection Agency, Tracor, Inc., Austin, Texas, February 1971.
2. FEDERAL WATER POLLUTION CONTROL ADMINISTRATION, "Delaware Estuary Comprehensive Study—Preliminary Report and Findings," Department of the Interior, Philadelphia, Pa., 1966.
3. CENTRAL PACIFIC BASINS COMPREHENSIVE WATER POLLUTION CONTROL PROJECT, "Effects of the San Joaquin Master Drain on Water Quality of the San Francisco Bay and Delta," U.S. Department of the Interior, Federal Water Pollution Control Administration, Southwest Region, San Francisco, Calif., January 1967.
4. DRONKERS, J. J., "Tidal Computations in Rivers and Coastal Waters," North-Holland Publishing Company, Amsterdam; U.S. publisher, John Wiley & Sons, Inc., New York, 1964.
5. MASCH, F. D., and R. J. BRANDES, Tidal Hydrodynamic Simulation in Shallow Estuaries, Rept. to Office of Water Resources Research, U.S. Department of the Interior, Tech. Rep. HYD 12-7102, Hydraulic Engineering Laboratories, University of Texas, Austin, August 1971.
6. THOMANN, R. V., Mathematical Model for Dissolved Oxygen, *Sanit. Eng. Div., Am. Soc. Civ. Eng.*, vol. 89, no. SA5, 1963.
7. PENCE, G. D., J. M. JEGLIC, and R. V. THOMANN, The Development and Application of a Time-Varying Dissolved Oxygen Model, Proceedings of the National Symposium on Estuarine Pollution, Stanford University, August 1967.

8. PENCE, G. D., and A. R. MORRIS, Quantitative Estimate of Migratory Fish Survival Under Alternative Water Quality Control Programs, Proceedings of the 5th International Conference on Water Pollution Research, Pergamon Press, New York, 1970.
9. THOMANN, R. V., "Systems Analysis and Water Quality Management," Environmental Science Service Publishing Co., Stamford, Conn., 1970.
10. HETLING, L. J., and R. L. O'CONNELL, A Study of Tidal Dispersion in the Potomac River, CB-SRBP Technical Paper No. 7, FWPCA, Region III.
11. HETLING, L. J., Simulation of Chloride Concentrations in the Potomac Estuary, CB-SRBP Tech. Paper No. 12, FWPCA Middle Atlantic Region, 1968.
12. HETLING, L. J., The Potomac Estuary Mathematical Model, Tech. Rept. No. 7, Chesapeake Technical Support Laboratory, FWQA, Annapolis, Md., 1969.
13. FEDERAL WATER QUALITY ADMINISTRATION, Problems and Management of Water Quality in Hillsborough Bay, Florida, Hillsborough Technical Assistance Project, Southwest Region, FWQA, 1969.
14. KAISER ENGINEERS et al., "San Francisco Bay-Delta Water Quality Control Program," Final Report to the State of California Water Resources Control Board, June 1969.
15. SHUBINSKI, R. P., J. C. MCCARTY, and M. R. LINDORF, Computer Simulation of Estuarial Networks, *J. Hydraul. Div., Am. Soc. Civ. Eng.*, vol. 91, no. HY5, 1965.
16. SHUBINSKI, R. P., and C. F. SHEFFEY, Wave Propagation in Estuarial Networks, Second Austral-Asia Conference on Fluid Mechanics, Auckland, New Zealand, December 1965.
17. ORLOB, G. T., Estuarial Systems Analysis—Quantity and Quality Considerations, Proceedings of the National Symposium on the Analysis of Water Resources Systems, American Water Resources Assn., Denver, Colo., July 1968.
18. ORLOB, G. T., R. P. SHUBINSKI, R. SELLECK, R. P. WALSH, and E. STANN, Modeling of Water Quality Behavior in an Estuarial Environment, Proceedings of the Fourth International Conference on Water Pollution Research, Prague, Czechoslovakia, April 1969.
19. ORLOB, G. T., and R. P. SHUBINSKI, Water Quality Modeling of Estuaries, Proceedings of the International Association on Scientific Hydrology, Conference on Hydrology of Deltas, Bucharest, Hungary, May 1969.
20. ORLOB, G. T., R. P. SHUBINSKI, and K. D. FEIGNER, Mathematical Modeling of Water Quality in Estuarial Systems, Proceedings of the National Symposium on Estuarine Pollution, Stanford University, 1967.
21. FEIGNER, K. D., and H. S. HARRIS, Documentation Report FWQA Dynamic Estuary Model, U.S. Department of the Interior, FWQA, July 1970.
22. CHEN, C. W., and G. T. ORLOB, Ecologic Simulation for Aquatic Environments, Annual Rept., OWRR Project No. C-2044, Office of Water Resources Research, U.S. Department of the Interior, August 1971.
23. SHUBINSKI, R. P., Analysis of the Effects of the Proposed Southern Crossing on the Hydraulic and Sedimentation Behavior of San Francisco Bay, Rept. to California Department of Public Works, Division of Bay Toll Crossings, San Francisco, June 1971.

24. CHEN, C. W., Concepts and Utilities of Ecologic Model, *J. Sanit. Eng. Div., Am. Soc. Civ. Eng.*, vol. 96n, SA5, 1970.
25. CHEN, C. W., An Ecologic Model for Water Resources Planning, Water Resources Engineering Conference, ASCE, Atlanta, Ga., January 1972.
26. REID, R. O., and B. R. BODINE, Numerical Model for Storm Surges in Galveston Bay, *J. Waterways Harbors Div., Am. Soc. Civ. Eng.*, vol. 94, no. WW1, pp. 33–57, 1968.
27. MASCH, F. D., et al., A Numerical Model for the Simulation of Tidal Hydrodynamics in Shallow Irregular Estuaries, Tech. Rept. HYD 12-6901, Hydraulic Engineering Laboratories, University of Texas, Austin, February 1969.
28. MASCH, F. D., et al., Tidal Hydrodynamic and Salinity Models for San Antonio and Matagorda Bay, Texas, Rept. to Texas Water Development Board, Austin, Texas, June 1971.
29. BRANDES, R. J., and F. D. MASCH, A Slowly-Varying Conservative Transport Model for Shallow Estuaries, Tech. Rept. HYD 12-7103, Hydraulic Engineering Laboratories, University of Texas, Austin, August 1971.
30. MASCH, F. D., M. NARAYANAN, and R. J. BRANDES, A Short-Term Conservative Transport Model for Shallow Estuaries, Tech. Rept. HYD 12-7104, Hydraulic Engineering Laboratories, University of Texas, Austin, Texas, August 1971.
31. DOUGLAS, J., and J. GUNN, A General Formulation of Alternating Direction Methods, *Numerical Math.*, vol. 6, 1964.
32. LEENDERTSE, J. J., Aspects of a Computational Model for Well-mixed Estuaries and Coastal Seas, RM 5294-PR, The Rand Corporation, Santa Monica, Calif., 1967.
33. LEENDERTSE, J. J., "A Water Quality Simulation Model for Well-mixed Estuaries and Coastal Seas," vol. I, Principles of Computation RM-6230-RC, The Rand Corporation, Santa Monica, Calif., 1970.
34. LEENDERTSE, J. J., and E. C. GRITTON, "A Water Quality Simulation Model for Well-mixed Estuaries and Coastal Seas," vol. II, Computation Procedures R-708-NYC, The Rand Institute, New York, July 1971.
35. LEENDERTSE, J. J., and E. C. GRITTON, "A Water Quality Simulation Model for Well-mixed Estuaries and Coastal Seas," vol. III, Jamaica Bay Simulation, R-709-NYC, The Rand Institute, New York, July 1971.
36. LEENDERTSE, J. J., Personal Communication January 1972.
37. MONSER, J. R., and D. E. EVENSON, An Investigation of the Effects of Waterfront Development Projects on East San Francisco Bay, Report to Interagency Council on Waterfront Problems, Albany, California, November 1971.
38. YOUNG, G. K., et al., Mathematical Modeling of Estuary Water Temperatures for Biscayne Bay, Report to National Park Service, U.S. Department of Interior, November 1971.