

## SURFACE-WATER QUALITY MANAGEMENT MODELS

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### 6-1 INTRODUCTION

In this chapter a variety of deterministic models will be examined for the preliminary selection of pollution-control policies for surface waters. Since Thomann,<sup>1</sup> Deininger,<sup>2</sup> Liebman,<sup>3</sup> Sobel,<sup>4</sup> and others introduced models for these purposes in the mid-1960s, there have been well over 50 models proposed for defining and evaluating regional water-quality management programs.<sup>5,6</sup> No attempt will be made to describe in detail each model or to discuss the various mathematical programming techniques that have been used to solve the models. This review will be limited to only the general framework of some of the recently proposed optimization models for defining and evaluating combinations of waste-water reduction and treatment, artificial aeration, flow augmentation, and bypass piping alternatives for the management of dissolved oxygen concentrations in regional surface-water systems. Readers interested in more detail on particular aspects of these and similar models may refer to Chap. 8 of this book and to many of the references provided at the end of this chapter.

As effective as optimization models are for the preliminary selection and analyses of water-quality management alternatives, clearly they by themselves

cannot accurately predict many of the important biological, chemical, and physical effects that will occur in water bodies given any particular management policy. Thus it is essential that other types of models be used to examine in more detail the results of these preliminary screening models before any specific policy is implemented. Simulation models (such as those discussed in Chap. 7) are well suited for such detailed analyses. Admittedly, water-quality simulation models are relatively more expensive to develop and solve than many optimization models; yet these simulation models do not require many of the assumptions and approximations that optimization models require in order to solve them. Consequently optimization models are best used, not necessarily to find the optimum solution, but to reduce the number of alternatives that need be simulated in the search of a satisfactory solution.

The emphasis in this chapter is on dissolved oxygen. While dissolved oxygen is an important and commonly used indicator of water quality, there are of course other important quality parameters, including, for example, various forms of nitrogen and phosphorus, numerous organic and inorganic chemicals and toxic industrial waste materials, pathogenic bacteria and viruses, pesticides and herbicides, sediment, and so on. All these potential pollutants are capable of being modeled, some even in a manner similar to the methods discussed in this chapter. The fact that many of these other water-quality parameters, at least until very recently, have not been modeled is mainly due to an insufficient amount of technical and economic data. Data are also lacking regarding the transfer of impurities between surface and subsurface or ground-water bodies. Because of this lack of information, it is usually assumed that the boundary between these two subsystems is impervious.

In spite of these and other important information limitations, water-quality management decisions must be made and are being made. This chapter reviews some of the preliminary screening models that have been proposed and used in conjunction with more detailed aquatic ecosystem simulation models to assist those involved in this decision-making process.

Those who develop water-quality management models usually assume the actual or potential existence of some governmental institution that has the authority to control water quality within its region, either by economic incentives such as effluent charges and/or by legal means such as effluent or stream standards. The need for such centralized control stems from the fact that water pollution is an externality that results, more often than not, in a diseconomy. Diseconomies occur when the use of a common property resource by one individual decreases the benefits derived from the use of that resource by other individuals. Water is an example of such a common property resource.

An individual polluter often has little or no incentive to reduce his own pollution activities; in fact, it may cost him a considerable sum of money to do so. Within the private market system, the damages a polluter incurs from his own pollution are often much less than it would cost him to reduce his pollution activities. There is no mechanism by which he is charged for the damages his pollution imposes on others. Yet society as a whole may benefit from improved

water quality. The main purpose of most regional water-quality models is to examine alternatives that will internalize and reduce both the private costs and public damages resulting from water pollution.

## 6-2 WATER-QUALITY PREDICTION AND CONTROL

### 6-2.1 Water-Quality Technology—A Brief Review

An important measure of quality in most water systems is dissolved oxygen. The dissolved oxygen concentration is a function of numerous parameters that describe the natural biochemical and physical processes which take place in streams, lakes, and estuaries. Desirable aquatic plant and animal life require oxygen for survival. Biodegradable wastes discharged into natural bodies of water are sources of food for many of the living organisms found in these natural waters. In order to utilize these wastes, these aquatic organisms require oxygen. They obtain their required oxygen from the oxygen that is dissolved in the water. Thus the decomposition of organic biodegradable wastes by aquatic organisms tends to reduce the dissolved oxygen concentration of the water that contains the wastes. The difference between the saturation concentration of dissolved oxygen and the existing dissolved oxygen concentration is called the *dissolved oxygen deficit*. Denoting DOS, DOC, and DOD as the dissolved oxygen saturation concentration, the actual dissolved oxygen concentration, and the dissolved oxygen deficit concentration, respectively, then by definition,

$$\text{DOD} = \text{DOS} - \text{DOC} \quad (6-1)$$

The oxygen required for the decomposition or assimilation of any particular quantity of waste is expressed as its *biochemical oxygen demand*, BOD. The oxygen demand of a waste can be separated into two components, the amount required for the assimilation of the carbonaceous waste material, BOD<sup>C</sup>, and the amount required for the assimilation of the nitrogenous waste material, BOD<sup>N</sup>. This division permits a more accurate description of the oxygen demand at any point in the stream, lake, or estuary, since the rates of deoxygenation associated with these two waste components usually differ. Another reason for explicitly considering the nitrogenous component is that as the efficiency of waste-water treatment increases the percentage of the nitrogenous component in the remaining waste-water effluent increases. With water-quality standards requiring increasingly higher waste removals or treatment efficiencies, the nitrogenous wastes discharged into natural waters become increasingly important for the prediction of dissolved oxygen concentrations.

The depletion of dissolved oxygen by the metabolic processes of waste-consuming organisms, plant respiration, benthic deposits, and the like, is offset by the absorption of oxygen from the atmosphere, from plant photosynthesis, and possibly from other natural and artificial means. Differential equations describing these processes of oxygen depletion and replacement are the basis of the water-quality control models described later in this chapter.

Water-quality models differ not only by the methods considered for quality control but also by the type of water body being polluted, by the choice of the water-quality management objective, and by whether steady-state or transitory solutions are desired. Regardless of these differences, the basis of all models for water-quality control is the fundamental principle of conservation of mass.

The conservation of mass principle applied to the concentration  $C(x,t)$  of a pollutant within any water body is simply stated. In any given interval of time, the mass transported into the water body minus the mass transported out of the body plus the mass produced (positive or negative) within the body equals the change in mass within that body during that time interval. On the basis of an analog of Fick's law of diffusion and the assumption of constant concentrations in both the lateral and vertical directions, a one-dimensional partial differential equation for conservation of mass can be written:

$$\frac{1}{A(x)} \left\{ \frac{\partial}{\partial x} \left[ D(x)A(x) \frac{\partial C(x,t)}{\partial x} \right] - \frac{\partial}{\partial x} [Q(x,t) \cdot C(x,t)] \right\} + R(x,t) = \frac{\partial C(x,t)}{\partial t} \quad (6-2)$$

where, in units of mass  $M$ , length  $L$ , and time  $T$ ,

$t$  = time  $T$

$x$  = a coordinate of a point on the water body  $L$

$A(x)$  = cross-sectional area at  $x$ ,  $L^2$

$D(x)$  = longitudinal dispersion coefficient at  $x$ ,  $L^2T^{-1}$

$C(x,t)$  = concentration of material of interest at  $x$  and  $t$ ,  $ML^{-3}$

$Q(x,t)$  = flowrate at  $x$  and  $t$ ,  $L^3T^{-1}$

$R(x,t)$  = net rate of addition (or subtraction) of material due to sources and sinks at  $x$  and  $t$ ,  $ML^{-3}T^{-1}$

The solution of this partial differential equation, subject to the appropriate boundary and initial conditions, represents the temporal and longitudinal distribution of the material of interest along a water course. As previously mentioned, the materials of interest in water quality include (but are certainly not limited to) both the carbonaceous and nitrogenous biochemical oxygen demand concentrations  $BOD^C$  and  $BOD^N$  and the dissolved oxygen concentration  $DOC$ . If the period of time is chosen so that both natural and waste-water flows are constant, a steady-state condition can be assumed. This implies that the concentration of a pollutant at a given point  $x$  does not change with time, i.e.,  $\partial C(x,t)/\partial t = 0$ . Only the concentrations of BOD and dissolved oxygen at various locations  $x$  along the water course need be determined. Clearly, the BOD and DOC concentrations do vary with time as well as distance, since flows, velocities, turbulence, and temperatures are not constant. However, for those water-quality control alternatives that are inflexible with respect to time, it is often reasonable to base the scale of these alternatives on some critical set of conditions that can occur at specified locations during certain times of the

day and year. Once these critical conditions are defined, only distance need be considered as variable.

In addition to the assumption that flows and pollutant concentrations do not change with time, if one can assume for the moment that the parameters for dispersion  $D(x)$ , flow  $Q(x)$ , and area  $A(x)$  are constant for all points  $x$  within some limited distance along a water course, then the steady-state form of Eq. (6-2) can be written

$$D \frac{d^2 C(x)}{dx^2} - \frac{Q}{A} \frac{dC(x)}{dx} + R(x) = 0 \quad (6-3)$$

To apply Eq. (6-3) to the estimation of BOD concentrations, it is necessary to define the production term  $R(x)$ . To do this it is assumed that the rate of change in BOD concentration ( $ML^{-3}$ ) with distance (velocity constant) is proportional to the BOD present and to the rate of BOD addition  $BR$  due to runoff and scour. Denoting the proportionality constant as a function of two parameters  $K_1$  and  $K_3$ , the deoxygenation and BOD sedimentation rate constants ( $T^{-1}$ ), respectively, Eq. (6-3) for BOD concentrations, either carbonaceous or nitrogenous, becomes

$$D \frac{d^2 \text{BOD}(x)}{dx^2} - \frac{Q}{A} \frac{d\text{BOD}(x)}{dx} - (K_1 + K_3) \text{BOD}(x) + BR(x) = 0 \quad (6-4)$$

The value of the deoxygenation and sedimentation rate constants  $K_1$  and  $K_3$  will depend on the composition of the BOD waste.

Equation (6-4) may be integrated to give the residual BOD at any distance or point  $x$  due to the steady discharge at another point  $x$ . If the waste source point is denoted as site  $i$  and the quality point is labeled site  $j$ , then the residual BOD at site  $j$  resulting from the discharge of BOD at site  $i$  can be simply defined as

$$\text{BOD}_{ij}^C = b_{ij}^C \text{BOD}_i^C \quad (6-5)$$

and

$$\text{BOD}_{ij}^N = b_{ij}^N \text{BOD}_i^N \quad (6-6)$$

where each constant  $b_{ij}$  is the number of units of BOD at site  $j$  resulting from the discharge of one unit of BOD at site  $i$ .

The total BOD at site  $j$  is simply the sum of both the carbonaceous and nitrogenous contributions from all waste source sites  $i$  plus the BOD resulting from the uncontrolled additions of BOD due to runoff, scour of bottom (benthic) deposits, decay of aquatic plants, etc. This uncontrolled contribution can be estimated by dividing the water course into a series of consecutive reaches  $r$  such that the rate of BOD addition  $BR_r$  is approximately uniform throughout the reach. The integration of Eq. (6-4) for each reach  $r$  yields the BOD at the end of each reach,  $\text{BOD}_r$ , that results from the uncontrolled additions,  $BR_r$ , within the reach. Each constant  $\text{BOD}_r$  is equivalent to a BOD input at a site  $r$  located at the end of reach  $r$ . This input multiplied by the constant  $b_{rj}$  is the BOD at site  $j$  downstream of reach  $r$  that results from the uncon-

trolled BOD additions  $BR_r$  within reach  $r$ . Hence the total BOD at each site  $j$  equals

$$\begin{aligned} \text{BOD}_j &= \sum_i (b_{ij}^C \text{BOD}_i^C + b_{ij}^N \text{BOD}_i^N) + \sum_r (b_{jr}^C \text{BOD}_r^C + b_{jr}^N \text{BOD}_r^N) \\ &= \sum_i (b_{ij}^C \text{BOD}_i^C + b_{ij}^N \text{BOD}_i^N) + \beta_j^C + \beta_j^N \end{aligned} \quad (6-7)$$

If the units of the BOD inputs,  $\text{BOD}_i$  and  $BR_r$ , are expressed in terms of concentrations ( $ML^{-3}$ ) or mass rates ( $MT^{-1}$ ), the resulting BOD at site  $j$  will be in similar units.

If the variables  $\text{BOD}_i$  and  $BR_r$  are in mass-rate units and  $\text{BOD}_j$  is desired in concentration units, the right-hand side of Eq. (6-7) can be multiplied by a unit conversion constant  $\alpha$ , if required, and then divided by the streamflow  $Q_j$  at site  $j$ :

$$\text{BOD}_j = \frac{\alpha}{Q_j} \left[ \sum_i (b_{ij}^C \text{BOD}_i^C + b_{ij}^N \text{BOD}_i^N) + \beta_j^C + \beta_j^N \right] \quad (6-8)$$

The use of Eq. (6-4) for estimating the carbonaceous and nitrogenous BOD concentrations at any site  $j$  does not require a constant cross-sectional area  $A(x)$ , a constant flow  $Q(x,t)$ , or a dispersion coefficient  $D(x)$  for all points  $x$  between sites  $i$  and  $j$ . What is required is that the average flows and areas (and, therefore, the velocities) and diffusion rates be defined for each pair of sites  $i$  and  $j$ . The same requirement applies to the estimation of dissolved oxygen using Eq. (6-3).

To determine the production  $R(x)$  term for dissolved oxygen, it is usually assumed that the rate of change in the dissolved oxygen concentration is proportional to the BOD present, the existing oxygen deficit, DOD, and to the rate of oxygen production ( $OR > 0$ ) or reduction ( $OR < 0$ ) due to plants, bottom deposits, and artificial aeration devices:

$$\begin{aligned} D \frac{d^2 \text{DOC}(x)}{dx^2} - \frac{Q}{A} \frac{d \text{DOC}(x)}{dx} + K_2 \text{DOD}(x) - (K_1^C \text{BOD}^C(x) \\ - (K_1^N \text{BOD}^N(x) + \text{OR}(x)) = 0 \end{aligned} \quad (6-9)$$

Here the parameter  $K_2$  is the reaeration rate constant ( $T^{-1}$ ), and the term  $OR$  is the rate of oxygen production ( $ML^{-3}T^{-1}$  or  $MT^{-1}$ ), its units depending on whether  $DOC$ ,  $DOD$ , and  $BOD$  are defined as concentrations ( $ML^{-3}$ ) or masses ( $M$ ), respectively. Equation (6-9) may be integrated to give the dissolved oxygen deficit  $\text{DOD}_{ij}$  at any site  $j$  resulting from the discharge of waste at site  $i$  and the uniform addition of BOD and/or oxygen between sites  $i$  and  $j$ :

$$\text{DOD}_{ij} = d_{ij}^C \text{BOD}_i^C + d_{ij}^N \text{BOD}_i^N - \delta_{ij}' \text{OR}_{ij} + \gamma_{ij}^C + \gamma_{ij}^N \quad (6-10)$$

Each coefficient  $d_{ij}$  is the oxygen deficit  $\text{DOD}$  at site  $j$  resulting from a single unit of  $\text{BOD}_i$  discharged at site  $i$ . Similarly each  $\delta_{ij}'$  is the quantity of oxygen at  $j$  corresponding to a unit input of oxygen  $\text{OR}_{ij}$  uniformly distributed

between sites  $i$  and  $j$ . The constant  $\gamma'_{ij}$  is the oxygen deficit at site  $j$  resulting from the addition of BOD,  $BR_{ij}$ , uniformly distributed between sites  $i$  and  $j$ . Again, to use Eq. (6-9) for estimating each deficit  $DOD_{ij}$ , the flow  $Q$ , area  $A$ , dispersion coefficient  $D$ , and reaeration rate constant  $K_2$  must reflect the average values between sites  $i$  and  $j$ . The units of the dissolved oxygen deficits  $DOD_{ij}$  will correspond to those of the inputs  $BOD_i$ ,  $BR_{ij}$ , and  $OR_{ij}$ , e.g., concentrations ( $ML^{-3}$ ) or mass rates ( $MT^{-1}$ ).

The total dissolved oxygen deficit at site  $j$  is simply the sum of all the individual deficits resulting from each controlled and uncontrolled waste discharge, oxygen sink, and oxygen source:

$$DOD_j = \sum_i (d_{ij}^C BOD_i^C + d_{ij}^N BOD_i^N) - \sum_r \delta_{rj} OR_r + \gamma_j^C + \gamma_j^N \quad (6-11)$$

The constant  $\delta_{rj}$  is the amount of oxygen at site  $j$  resulting from a unit of oxygen  $OR_r$  added uniformly along reach  $r$ , and  $\gamma_j$  is the total oxygen deficit at site  $j$  resulting from the uncontrolled addition of BOD,  $BR_r$ , within each reach  $r$  of the water course.

The dissolved oxygen concentration at site  $j$  is simply the saturation concentration  $DOS_j$  minus the deficit concentration  $DOD_j$ . If the deficit is expressed as a mass rate ( $MT^{-1}$ ), then multiplying it by unit conversion constant  $\alpha$  divided by the flow  $Q_j$  ( $L^3T^{-1}$ ) at site  $j$  will result in the appropriate concentration units:

$$DOC_j = DOS_j - \frac{\alpha}{Q_j} DOD_j \quad (6-12)$$

For nontidal streams the dispersion coefficient  $D(x)$  in the above equations is usually an order of magnitude smaller than for tidal streams. In addition, the second derivative of the longitudinal concentration profile is small; thus the first term of Eqs. (6-4) and (6-9) may be ignored, leaving the well-known Streeter-Phelps equations expressed in terms of distance rather than time of flow. For estuaries, alternatives to the integration of Eq. (6-9) for estimating the values of the transfer coefficients  $d_{ij}$ ,  $\delta_{ij}$ , and  $\gamma_j$  are available and will be discussed later. Equations (8-25) to (8-27) define in greater detail some of the constants used in Eqs. (6-5) to (6-8) and (6-10) to (6-12).

Before structuring a variety of models for managing river-water quality, it might be helpful to discuss some of the alternatives and objectives often considered in the development of water-quality control policies.

### 6-2.2 Alternative Methods for Water-Quality Control

The first and most obvious method of water-quality control is that of limiting the amount of waste discharged into receiving bodies of water. This type of control can take on numerous forms, some of which include:

- I Requirements that each waste producer produce less waste through,

for example, process changes and/or remove at least some minimum specified fraction or percentage of his BOD prior to discharging the remainder into natural waters. Removal of BOD, i.e., waste-water treatment, is accomplished by a variety of physical, biological, and chemical processes.<sup>7,8</sup>

2 Storing that portion of the treated waste-water effluent, which if released into the natural water body would result in a lower-than-desired quality. Either ponds or tanks could be used for effluent storage. The quantity and timing of stored effluent discharges would depend in part on the waste assimilative capacity of the receiving water body. The greater the variability in either effluent waste concentrations and/or waste assimilative capacities, the more attractive this particular alternative becomes.

3 Piping wastes, either prior to or subsequent to some treatment, to areas within or outside of the region for additional treatment and/or disposal at sites having greater waste assimilative capacities. This alternative also permits the central processing of wastes from many areas, taking advantage of economies of scale in construction and operating costs, as well as of increased operating efficiencies resulting from being able to hire operators with increased skills.

A second method of controlling water quality is the use of artificial aeration devices.<sup>9</sup> These devices transfer oxygen, usually from the air, into the water body by methods that include air injection through a network of perforated pipes or aeration through surface agitation. Oxygen quantities added to the water at any specified location depend not only on the oxygen available but also on the existing oxygen deficit. The oxygen added in this manner may be varied in distance and time, depending on the varying amounts required to meet water-quality standards.

A third method of controlling water quality is to increase the dilution and assimilation of wastes through flow augmentation during critical periods in the year.<sup>10,11</sup> Since it is not the absolute amount of wastes or oxygen in the water, but the concentration of wastes, that determine water quality, flow augmentation during periods of low natural flows may be an effective means of increasing water quality.

### 6-3 QUALITY OBJECTIVES AND STANDARDS

A perfectly reasonable efficiency objective for any particular watershed, given levels of economic output and consumption, might be the minimization of the costs of water-quality improvement alternatives plus all external damages associated with the resulting water quality. Because the private market system fails to charge each polluter an amount equal to the damages resulting from his waste discharge, governmental action is often required to assist in achieving a



minimum level of costs and damages associated with water quality. The types of incentives that water-quality regulatory agencies have used to compensate for the failure of individual polluters to consider the damages they impose on others are (1) *legislative*, including direct regulation, the establishment of effluent or stream-quality standards, licensing, and zoning; (2) *legal*, including compensation for damages and fines for violation of law; and (3) *economic*, including effluent charges or taxes, subsidies, accelerated depreciation allowances, and the like. Whatever the methods used, the objective is to achieve a more efficient and equitable allocation of resources from the standpoint of society as a whole.

One of the difficulties in achieving an objective that is both efficient and equitable is the problem of quantifying water-quality benefits or damages. The problems are similar to those of attributing a dollar benefit to such things as aesthetics and clean air. There is also the problem of determining equitable distributions of costs and benefits. Thus, the selection of the desired water quality and the question of who will pay for it often becomes a political decision. This political aspect is reflected in both the water-quality management objective and in the quality standards. The political system has clearly demonstrated its extreme sensitivity to the unquantifiable aspects of water-quality management problems.

### 6-3.1 Water-Quality Objectives

To begin quantifying a rather general objective for water-quality management, consider a river basin in which there are numerous individuals or groups  $i$  that discharge pollutants into the natural water courses. Included among those individuals or groups of individuals are organizations such as state and federal pollution-control agencies that have interests, financially as well as politically, in the quality of the natural water within the basin. Water-quality control alternatives such as waste-water treatment and effluent storage impose costs to private and, because of cost-sharing programs, state and federal agencies as well. Nonreimbursable quality-control alternatives such as flow augmentation and artificial aeration may only add to the cost paid by state and federal agencies.

Regardless of who pays, the cost to each individual or group  $i$  can, for the moment, be denoted as a function of the scale of all alternatives used for future quality control.

The quantity of the wastes discharged at any site is a function of the scale of the alternatives used for reducing the wastes at that site. Clearly these waste-reduction alternatives should include long-run changes in the production and consumption of various goods and services, as well as more short-run alternatives such as waste-water treatment.

Denoting  $S$  as the scale of some set of waste-reduction alternatives costing  $C_i(S)$  for each group  $i$ , a cost-effective objective without regard to cost dis-

tribution, or the political influence of each group  $i$ , can be written

$$\text{Minimize} \quad \sum_i C_i(\mathbf{S}) \quad (6-13)$$

in which the resulting quality must be equal to or greater than the minimum allowable quality established by law. Such an objective may or may not result in a socially efficient solution. This depends in part on the quality standards imposed. Usually the minimum allowable quality standards are not intended to represent the desired quality. Environmental protection goals and allowances for future growth and uncertainties often result in target or desired qualities that are higher than specified minimum quality standards. One way to achieve a quality that comes closer to the target is through the proper use of effluent charges or taxes  $T_i$  based on the waste released by each group or at each site  $i$ .

The fraction  $P_i = P_i(\mathbf{S}_i)$  of the waste reduced or removed at each site  $i$  is, of course, a function of the scale  $\mathbf{S}_i$  of waste-reduction measures employed at each of those sites. If  $\text{BOD}_i$  is the quantity of BOD available at site  $i$  prior to the implementation of any BOD reduction measures, then  $\text{BOD}_i(1 - P_i)$  is the remaining quantity that will be discharged into the water body. Ideally any tax on the amount of waste discharged should reflect the external damages attributable to that residual waste discharge. Of course, such taxes can be either positive or negative if subsidies are appropriate. Clearly the purpose of the tax or subsidy is to provide an economic incentive for reducing the external damages, if any, that result from the discharge of wastes.

Including the effluent tax or subsidy in the criterion function, the objective of the discharger becomes one of minimizing the total cost  $C_i(\mathbf{S})$  of all alternatives  $\mathbf{S}$  used to reduce the quantity of waste discharged plus the external damages  $T_i[\text{BOD}_i(1 - P_i)]$  resulting from the discharged wastes  $\text{BOD}_i(1 - P_i)$ ,

$$\text{Minimize} \quad \sum_i \{C_i(\mathbf{S}) + T_i[\text{BOD}_i(1 - P_i)]\} \quad (6-14)$$

with or without some constraints on water quality.

Note that the above objective does not attempt to quantify the benefits or damages associated with the resulting water quality, except through the establishment of effluent charges and perhaps quality standards. These charges and standards would have to be defined prior to their incorporation into water-quality models. One of the advantages, however, of model construction and solution prior to the final establishment of charges and standards is that of being able to estimate the costs to each group  $i$  and the resulting water quality associated with various proposed combinations of charges and standards.

Whether or not additional components such as water-quality benefits or damages (if quantification is possible) or water treatment costs are included in the model,<sup>12</sup> the objective function as defined describes the central problem in water-quality control, namely, the trade-off between the benefits (reduced taxes) derived from higher water quality and the cost of producing that higher

quality. As it stands, however, the objective is still incomplete. To be politically acceptable it must reflect the fact that it is not only the total minimum cost plus tax which is important; it is also who pays these costs and taxes. *Who* pays often counts as much as if not more than *how much* is paid.

The political process of establishing effluent charges and minimum acceptable qualities, either in the form of effluent or stream-quality standards, involves the participation of each group of individuals within the river basin. Some groups clearly have more political influence than others. This depends not only on their political power but also on how strongly they feel about certain issues. To include the effect of this political influence in water-quality models, it is often assumed that relative weights can be defined and used in the objective function. Each weight  $W_i$  reflects the relative influence that group  $i$  exerts compared to all other groups defined by the model. Using the proper weights, charges, and standards, a socially or politically equitable and efficient water-quality management policy would result from the following objective:

$$\text{Minimize} \quad \sum_i W_i \{C_i(S) + T_i[BOD_i(I - P_i)]\} \quad (6-15)$$

The difficulty here, of course, is that the relative political weights are unknown, even to the decision makers, until the final decision is made. Without a knowledge of the benefits or damages associated with water quality and of the actual relative political weights, it is unlikely that the chosen effluent charges, weights, and quality standards will result in a socially efficient and equitable solution. However, for any assumed set of weights, charges, and standards, the solution of any model having the above objective can be termed a politically effective solution.

By varying the relative weights, an analyst can define a set of politically effective alternatives from the infinite set of possible alternatives. If the objective is piecewise linear, the number of effective alternatives will be smaller than the number of possible combinations of relative weights, and certainly much smaller than the total number of possible alternatives. In some cases it may be possible to choose weights that will define all possible effective alternatives. This exhaustive procedure may be avoided by developing some subjective probabilities for various sets of weights and examining only those sets whose acceptance is most probable. Clearly, such an approach is predictive rather than normative.

If all the relative weights are assigned values of unity, the objective function (6-15) is identical to the function (6-14) and will represent the minimization of total costs and damages without regard to any redistribution of these costs and damages among various polluters. As the relative weights change, so will the alternatives associated with those weights. Relatively high weights will correspond to those polluters having a relatively strong political position and interest, which in effect will reduce their share of the total cost. Varying the weights permits an examination of the stream quality that is likely to be associated with various cost distributions.

Before concluding this discussion of political weights, it should be pointed out that most systems analysts are aware of the fact that few, if any, decision makers want to be placed in a position of saying that one polluter should be favored, say, three times as much as another, and so on. Many will not want to know what the weights are even after they have made their decision. Nevertheless, their decision reflects the relative weights given to each individual or objective. Thus, while analysts cannot pretend to know which set of weights will ultimately be chosen, they can use them to advantage in defining and evaluating various politically effective alternatives that decision makers can consider during the decision-making process, with or without knowledge of the weights used to generate them.

Other means have been used to incorporate considerations of equity into otherwise strictly cost-effective models [e.g., those having objective functions of the type (6-13)]. These include constraints specifying equal scales of various alternatives, or some function of these scales such as equal treatment efficiencies, or equal costs per capita contributing to the total waste at various sites. If  $S_i$  is the scale of quality control alternatives employed at site  $i$ , then constraints requiring equal scales of control could be written as

$$S_i = S_k \quad \forall i \in I_k \quad (6-16)$$

where  $I_k$  is the set of sites  $i$  in zone  $k$  of the region. Zones within a region may be defined geographically or by types of polluters.

Although numerous water-quality systems analysts have included equity within the constraint set of their models, it could be argued that equity is an objective—one of many that water-quality planners consider. The relative weight given to an equity objective depends in part on the economic cost of achieving it, as well as on the administrative and political cost of not achieving it, i.e., the unquantifiable cost associated with implementing a plan calling for a wide range of quality control requirements within a region that minimizes only the total economic costs. Equity within any zone of a river basin can be expressed as an objective by defining and then minimizing the absolute difference between the minimum and maximum scale of water-quality control within the zone. Let  $S_k^{\min}$  and  $S_k^{\max}$  denote variable lower and upper bounds of the scale of a single control alternative within zone  $k$  so that

$$S_k^{\min} \leq S_i \leq S_k^{\max} \quad \forall i \in I_k \quad (6-17)$$

A portion of the equity objective can involve minimizing the sum of the weighted differences between  $S_k^{\min}$  and  $S_k^{\max}$  over all zones  $k$ :

$$\text{Minimize} \quad \sum_k W_k (S_k^{\max} - S_k^{\min}) \quad (6-18)$$

Defining a relative weight  $W_{kl}$  denoting the relative importance of the difference between the minimum, average, or maximum scale of control in zone  $k$  and the corresponding scale of control in zone  $l$  permits the inclusion of equity among different zones in the multiple-objective function. How this can be done

is illustrated by Eqs. (6-19) and (6-20), using the maximum scale of control in each zone  $k$  ( $k = 1, 2, \dots, K$ ):

$$\text{Minimize} \quad \sum_{k=1}^{K-1} \sum_{l>k}^K W_{kl} (s_{kl} + s_{lk}) \quad (6-19)$$

where, for each pair of zones  $k$  and  $l$ ,  $s_{kl} \geq 0$ ,  $s_{lk} \geq 0$ , and

$$S_k^{\max} - S_l^{\max} = s_{kl} - s_{lk} \quad (6-20)$$

Objective (6-19) ensures that, if  $s_{kl} > 0$ ,  $s_{lk} = 0$ , or if  $s_{lk} > 0$ ,  $s_{kl} = 0$ .

### 6-3.2 Water-Quality Constraints

Constraints on water quality can be defined as effluent standards, restricting the BOD released by group  $i$ ,  $BOD_i(1 - P_i)$ , to be no greater than the maximum allowable quantity

$$BOD_i(1 - P_i) \leq BOD_i^{\max} \quad (6-21)$$

or stream-quality standards requiring the quality  $Q_j(S)$  at a site or reach  $j$  within the water body to be no less than some minimum allowable quality

$$Q_j(S) \geq Q_j^{\min} \quad (6-22)$$

These objectives and constraints will be defined in greater detail below.

## 6-4 RIVER-QUALITY CONTROL MODELS

In each of the models that follows, waste-water treatment and/or reduction through process changes, i.e., the removal of some fraction of the total waste load prior to discharging the remainder into the receiving water bodies, will be included as a method of controlling water quality. At first, only waste-water treatment and reduction alternatives will be considered. Following this discussion other methods of water-quality control will be introduced and incorporated into models for defining and evaluating combinations of these alternative methods for managing water quality.

### 6-4.1 Waste-Water Treatment and Reduction Models

In this section waste-water treatment and/or process changes that reduce the quantity of BOD produced prior to waste-water treatment will be the only two alternatives considered. The problem is one of estimating the fraction of waste to be reduced at each waste source site  $i$  that will provide the desired water quality and at the same time satisfy some management objective.

The cost to each waste maker  $i$  results from the treatment or reduction in the production of waste and, if applicable, a charge based on his discharged

waste effluent. Reduction costs are commonly expressed in terms of the fraction of carbonaceous BOD removed. Yet, associated with a given fraction,  $P_i^C$ , of carbonaceous BOD removal is a specific fraction  $P_i^N$  of nitrogenous BOD removal. Since both types of BOD are reduced simultaneously,  $P_i^N$  is some function of  $P_i^C$ . Hence, the cost of waste-water reduction involving both carbonaceous and nitrogenous BOD removal can be expressed as a function of  $P_i^C$  or  $P_i^N$ . Here,  $C_i(P_i^C)$  will denote the total annual reduction cost at each waste source site  $i$ .

An effluent tax or subsidy  $T_i$  may be a function of the quantity of both types of BOD (and, in fact, many other potential pollutants as well) that are discharged into the stream. If  $BOD_i^C$  and  $BOD_i^N$  are the carbonaceous and nitrogenous BOD concentrations prior to reduction at each site  $i$ , then the quantities discharged into the stream or estuary after reduction equal  $BOD_i^C(1 - P_i^C)$  and  $BOD_i^N(1 - P_i^N)$ , respectively. An effluent charge on these discharged wastes will be denoted by the function  $T_i[BOD_i^C(1 - P_i^C), BOD_i^N(1 - P_i^N)]$ . The objective function (6-19) can now be written

$$\text{Minimize } \sum_i W_i \{C_i(P_i^C) + T_i[BOD_i^C(1 - P_i^C), BOD_i^N(1 - P_i^N)]\} \quad (6-23)$$

This objective can be minimized subject to both effluent standards at each waste source site  $i$ :

$$BOD_i^C(1 - P_i^C) + BOD_i^N(1 - P_i^N) \leq BOD_i^{\max} \quad (6-24)$$

and/or stream-quality standards for each quality site  $j$ :

$$BOD_j \leq BOD_j^{\max} \quad (6-25)$$

$$DOC_j \geq DOC_j^{\min} \quad (6-26)$$

Equations (6-24), (6-25), and (6-26) are similar in form to Eqs. (6-21) and (6-22) above. To obtain the BOD concentration at a quality site  $j$ ,  $BOD_j$ , Eq. (6-8) is used after multiplying the  $BOD_i$  terms by  $(1 - P_i)$  to compensate for waste-water removal or reduction at site  $i$ . To obtain the dissolved oxygen deficit at site  $j$  resulting from all waste discharges at site  $i$ , equations similar to (6-10) and (6-11) or to (6-12) are used. Equation (6-11) defines the total dissolved oxygen deficit at  $j$  resulting from the input of BOD at all sites  $i$ . Incorporating waste removal or reduction alternatives in Eq. (6-11), the dissolved oxygen deficit at site  $j$  equals

$$\begin{aligned} DOD_j = \sum_i [d_{ij}^C BOD_i^C (1 - P_i^C) + d_{ij}^N BOD_i^N (1 - P_i^N)] \\ - \sum_r \delta_{rj} OR_r + \gamma_j^C + \gamma_j^N \end{aligned} \quad (6-27)$$

where, here, it is convenient to assume that both BOD inputs and the resulting dissolved oxygen deficit are expressed as mass rates ( $MT^{-1}$ ). Given the dissolved oxygen deficit  $DOD_j$  for all sites  $j$ , the dissolved oxygen concentration

at  $j$ ,  $DOC_j$ , from Eq. (6-12), equals

$$DOC_j = DOS_j - \frac{\alpha}{Q_j} DOD_j \quad (6-28)$$

where  $\alpha$  is a constant converting the units of  $DOD_j$  divided by the flow  $Q_j$  at site  $j$  into the appropriate units of concentration. Again  $DOS_j$  is the dissolved oxygen saturation concentration at site  $j$ .

Additional constraints defining  $P_i^N$  as a function of  $P_i^C$  and limiting the range of  $P_i^C$  are also required:

$$P_i^N = f_i(P_i^C) \quad \forall_i \quad (6-29)$$

$$0 \leq P_i^C \leq 1.0 \quad \forall_i \quad (6-30)$$

There are several aspects of this waste-water treatment and reduction model worth emphasizing. First, note that the quality is constrained only at specified sites  $j$  along the stream. Yet, for the assumed values of the exogenous variables, solutions of this model are technically acceptable only if at all points in the stream, the dissolved oxygen concentration  $DOC$  is greater than zero. Hence, a sufficient number of quality sites  $j$  must be selected to ensure this condition.

In addition to the above, there are other reasons why stream-quality standards are specified for whole reaches or sections of streams. Such standards are necessary to maintain adequate conditions for fish and other desirable aquatic life. Few benefits are derived from ensuring sufficient dissolved oxygen for fish spawning at upstream sites if the fish can survive neither the swim to nor from those sites because of excessive oxygen deficits enroute. If reaches are defined so that the parameters affecting the BOD or dissolved oxygen concentrations at specified locations in each reach are relatively constant, then linear constraints, limiting the BOD concentration at the beginning of each reach  $r$  ( $BOD'_r$ ) to a function of both the quality standard in that reach and the initial dissolved oxygen concentration ( $DOC_r$ ), can be specified to ensure that at least some minimum concentration within the reach will be maintained:

$$BOD'_r \leq \alpha_r DOC_r + \hat{\gamma}_r \quad (6-31)$$

The values of the coefficients  $\alpha_r$  and  $\hat{\gamma}_r$  can be obtained by methods described elsewhere.<sup>13,14</sup> These methods turn out to be somewhat involved, and for most practical purposes a few trial runs of the model will indicate where quality sites  $j$  will have to be located to insure a minimum dissolved oxygen concentration equal to or greater than the quality standard throughout each reach of the stream.

Another aspect of this waste-water treatment and reduction model deserving some discussion is its deterministic character. Waste-water influents usually vary in both flow and BOD concentration;<sup>15</sup> hence, the waste-water effluent quality can be expected to vary. Streamflows are also variable. For the

purposes of planning waste-water treatment efficiencies, some "design" influent flow and BOD concentration must be selected, e.g., that which is exceeded only 5 percent of the time. Similarly, some "design" streamflow  $Q_j$  at each quality site  $j$  must be chosen. The streamflow  $Q_j$  often selected in the more humid areas of the United States is the minimum average 7-day consecutive flow expected once every 10 years. This is an extremely low streamflow, often being exceeded more than 99 percent of the time.<sup>16</sup> The selection of a low streamflow for  $Q_j$  is based on the assumption that higher streamflows will result in higher dissolved oxygen concentrations at any particular quality site. This assumption is not always valid, as will be discussed later when low flow augmentation alternatives are examined.

#### 6-4.2 Thermal-Loading Control Models

The above model for estimating effective treatment and reduction alternatives can be extended in a number of ways. Dysart<sup>17</sup> and others have considered the discharge of thermal energy as a specific pollutant and have developed models for examining the effect of heat on the BOD and DOC concentrations.

The dissipation of water temperature can be defined by an exponential decay function resulting from the integration of the differential equation

$$\frac{Q}{A} \frac{d(T^\circ - T_e^\circ)}{dx} = -K_T (T^\circ - T_e^\circ) \quad (6-32)$$

in which  $T^\circ$  is the existing water temperature and  $T_e^\circ$  the equilibrium water temperature. The constant  $K_T$  is the temperature dissipation rate coefficient ( $T^{-1}$ ).

Water temperature affects the saturation dissolved oxygen concentration DOS as well as the deoxygenation and reaeration rate constants  $K_1$  and  $K_2$ . An increase in water temperature results in increased rate constants  $K_1$  and  $K_2$  and in reduced dissolved oxygen saturation concentrations. The variation in rate constants  $K_1$  and  $K_2$  with temperature can be estimated from the following two equations in which 20°C is taken as a base temperature:

$$K_1(T^\circ) = K_1(20^\circ) [1.047]^{(T^\circ - 20^\circ)} \quad (6-33)$$

$$K_2(T^\circ) = K_2(20^\circ) [1.024]^{(T^\circ - 20^\circ)} \quad (6-34)$$

Since the increase in  $K_1$  is greater than  $K_2$ , the overall effect of a rise in water temperature is an increase in the maximum deficit DOD and a shortening of the distance over which the dissolved oxygen deficit resulting from the discharge of a particular pollutant occurs.

Alternatives for the "treatment" or reduction of thermal energy in effluents include the use of cooling ponds and towers or internal process changes within the heat-producing plant. Cost functions in terms of the scale of such alternatives could be included in objective functions of the type (6-23). In these cases the constraint set would include functional relationships between the scale of any heat-reducing alternative and the amount of heat removed. For



example, the fraction of heat removed in a waste-water effluent, similar to that used for BOD removal, could be the measure of the scale of any particular alternative.

Constraints on the quality of heat discharged into water bodies are of a form similar to those limiting BOD and DOD concentrations. Both effluent standards, specifying the maximum rise in temperature some distance from the point of discharge, or stream standards, specifying the maximum allowable temperatures at various distances along the water course, can be used.

### 6-4.3 Flow Augmentation Models

The models discussed above are all based on a critical or design flow, denoted as  $Q_j$  at each quality site  $j$ . Given a constant discharge of BOD from one or more upstream sources, the dissolved oxygen deficit at various quality sites downstream may either decrease or increase, depending on the quantity and quality of the streamflow. For example, consider a single waste source site upstream from some quality sites. If the DOD and BOD concentrations of the streamflow upstream of the waste discharge site are less than the respective DOD and BOD concentrations in the discharged waste water, and if the increased streamflows do not pick up additional BOD from increased runoff and scour of bottom deposits, then the additional dilution of the waste-water flow will decrease the BOD concentration and therefore increase the minimum dissolved oxygen concentration downstream from the waste discharge site. While the minimum dissolved oxygen concentration is increased, these same conditions may lower the dissolved oxygen concentration at one or more specific quality sites. Such a situation is illustrated in Fig. 6-1.

In Fig. 6-1 each pair of functions correspond to the oxygen "sag" concentration [Eq. (6-9)] and the BOD concentration [Eq. (6-4)] resulting from a single waste source at site  $i = 0$  given three streamflows  $Q_1 < Q_2 < Q_3$  at each quality site  $j$ . Note that, for the conditions stated above, the minimum dissolved oxygen concentration increases as the streamflow increases. At quality site  $j = 1$ , increases in the streamflow result in decreased dissolved oxygen deficits and BOD concentrations; i.e., the water quality improves with increasing streamflows. The opposite may occur at quality sites  $j = 2$  and 3. At these sites the quality decreases as the streamflow increases, at least up to a certain quantity. Hence there can exist situations, as illustrated in Fig. 6-1, in which the waste-removal efficiencies at upstream waste-water treatment facilities designed to meet both dissolved oxygen and BOD stream-quality standards at low streamflows (e.g., the minimum average 7-day consecutive streamflow expected once in 10 years) are not sufficient to meet these same quality standards at higher streamflows. Since streamflows in excess of the commonly chosen low "design" flows occur much more frequently, there may be a considerable portion of the time when the stream quality at particular downstream sites is less than that desired. The selection of the critical "design" flow  $Q_j$  therefore

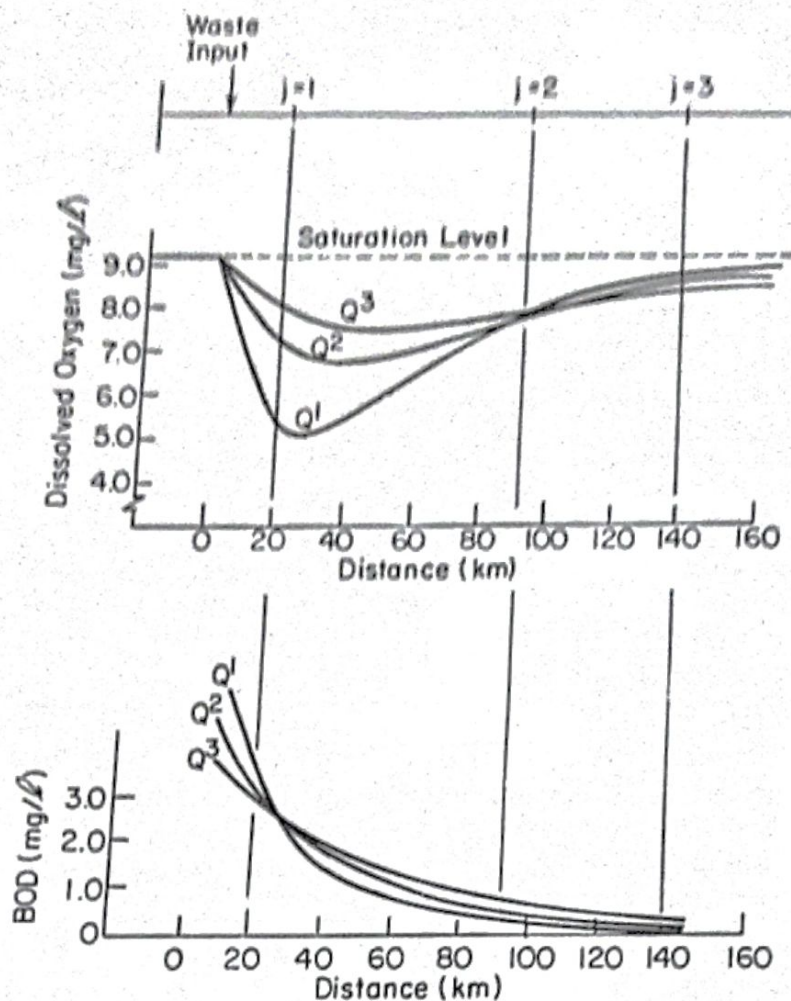


FIGURE 6-1  
Dissolved oxygen and BOD concentrations downstream from a single waste source under increasing flow conditions ( $Q_1 < Q_2 < Q_3$ ).

becomes an important consideration in the determination of treatment-facility waste-removal efficiencies.

The critical streamflows  $Q_j$  can be augmented or increased by releasing additional waters from reservoirs, or by reducing water consumption, upstream of any quality site  $j$ . These additional flows may be a means of decreasing the critical flow conditions (e.g., increasing the flow in the stream or estuary, reducing temperatures, increasing the reaeration rates, etc.), thereby reducing required treatment costs.

Flow augmentation has three primary effects on water quality. First, augmenting flow increases the volume of water in a water body. As already mentioned, if the augmenting flow is of higher quality than the base flow, the

increased flow increases the minimum dissolved oxygen concentration. Second, flow augmentation increases the water velocity, which in turn usually increases the reaeration rate and lengthens the distance over which a pollutant causes an oxygen deficit. Third, if base flows are augmented with cooler water, the deoxygenation rate decreases and the saturation concentration of dissolved oxygen increases. Conversely, if higher temperature waters are used for augmentation, deoxygenation rates increase and saturation dissolved oxygen concentrations decrease. Finally, increased streamflows may increase the BOD addition due to runoff and scour of benthic deposits. All these factors may well result in flow augmentation being beneficial to some and detrimental to others. The complexity of this water-quality alternative can only be outlined in the discussion that follows. For further detail see, for example, the studies of Pyatt et al.,<sup>10</sup> Jaworski, Weber, and Deininger,<sup>18</sup> and Loucks and Jacoby.<sup>11</sup>

The net costs of flow augmentation can be defined as the minimum costs necessary to increase the flows during the period of low flows for the sole purpose of water-quality management. In other words, the cost of augmentation is that required over and above the costs needed to maximize the net benefits of flow regulation for nonquality uses. If such optimal conditions do not exist at the time flow augmentation is being considered, then the benefits derived from flow augmentation, apart from improved quality, should be subtracted from the gross costs of augmentation. This net cost  $C_Q(\Delta Q)$  of the flow increase  $\Delta Q$  can then be added to the cost-minimization objective function (6-23). In the United States, flow augmentation is currently a nonreimbursable federal cost, and thus there are no cost-allocation problems that require solution. Numerous schemes could be proposed should the nonreimbursable aspect of this cost be modified in the future.

Because the reaeration rate constants  $K_2$  and the time of flow from any waste discharge site  $i$  to quality site  $j$  change as the flow is increased, the coefficients  $d_{ij}(Q_{ij})$ ,  $\beta_{ij}(Q_{ij})$ , and  $\delta_{ij}(Q_{ij})$  of Eq. (6-8) for defining  $BOD_j$  and Eq. (6-11) for defining  $DOD_j$  are now functions of the variable flows  $Q_{ij}$  between sites  $i$  and  $j$ . For any flow  $Q_j + \Delta Q_j$  at site  $j$  and upstream treatment efficiencies  $P_i$ , the dissolved oxygen concentration at each site  $j$  equals

$$DOC_j = DOS_j - \frac{\alpha}{Q_j + \Delta Q_j} DOD_j \quad (6-35)$$

Similarly, the total BOD concentration at each site  $j$  is

$$BOD_j = \frac{\alpha}{Q_j + \Delta Q_j} \left[ \sum_i (b_{ij}^C BOD_i^C + b_{ij}^N BOD_i^N) + \beta_j^C + \beta_j^N \right] \quad (6-36)$$

Substituting these  $DOC_j$  and  $BOD_j$  variables into constraint equations (6-25) and (6-26) completes the inclusion of flow augmentation as a water-quality control alternative in addition to waste-water treatment.

#### 6-4.4 Artificial Aeration Models

Another method of improving water quality is that of artificial aeration. This alternative is usually accomplished by injecting oxygen (air) into the water through a network of perforated pipes or by rotating devices that cause surface turbulence, thereby increasing the area over which oxygen transfer can occur. These methods may be particularly efficient for the temporary improvement of near-anaerobic conditions; i.e., at sites where the dissolved oxygen deficits are relatively high.

The oxygen transfer rate due to aeration devices is usually expressed as a mass rate per unit of power input, e.g., milligrams per horsepower-hour. This rate varies directly with the oxygen deficit, the water quality, the temperature, and the flow. Ortolano<sup>19</sup> has proposed the following relationship for the prediction of the rate of oxygen transfer through artificial aeration at any site  $i$ , given the dissolved oxygen deficit  $DOD_i$  at that site:

$$H_i = H_s k_i (DOD_i) \quad (6-37)$$

In the above equation the variable  $H_i$  is the rate of oxygen transfer per unit of power input ( $MT^{-1}/ML^2T^{-3} = L^{-2}T^2$ ) under field conditions,  $H_s$  is the rate of oxygen transfer under standard conditions (i.e., tap-water quality at 20°C, zero DOC, and 1 atmosphere of pressure),  $DOD_i$  is the dissolved oxygen deficit concentration at site  $i$ , and  $k_i$  is a function of the actual field conditions at site  $i$ :

$$k_i = \frac{\mu \lambda^{(T^{\circ}-20)}}{9.2} \quad (6-38)$$

The parameters  $\lambda$  and  $\mu$  are two dimensionless empirical correction factors to account for the dependence of the oxygen transfer rate on temperature and on water quality, respectively. Assuming  $H_i$  is an average value of the oxygen transfer throughout the unit of time used in an analysis of water quality (usually a tidal cycle for estuaries and a day for streams so that the change in oxygen concentration at a particular site  $i$  with respect to time can be assumed equal to zero), then the oxygen transfer rate  $OR_i$  ( $MT^{-1}$  or  $ML^{-3}T^{-1}$ ) at site  $i$  from artificial aeration is the product of the weight of oxygen per power input unit  $H_i$ , the total power input per unit of time,  $W_i$  ( $ML^2T^{-3}$ ), and a coefficient  $\zeta$  to convert the product into the appropriate units as required for incorporation into Eq. (6-10):

$$OR_i = \zeta H_i W_i = \zeta H_s k_i DOD_i W_i \quad (6-39)$$

An alternative approach is to include artificial aeration alternatives within models that are not steady state; i.e., those which predict both the dissolved oxygen and BOD concentrations as a function of both time and distance (or location). Either approach properly assumes a flexible procedure for the operation of the aeration system; i.e., the system can be used only when needed during the critical periods of the year.

The capacity of artificial aerators is usually expressed in horsepower units. Clearly, the oxygen transferred per unit of time  $OR_i$  at each site  $i$  cannot exceed the capacity of the aerator  $A_i$  times the total weight transferred per unit power input  $H_i$ ,

$$OR_i \leq \zeta H_i A_i \quad (6-40)$$

which is equivalent to specifying that the power input per unit of time  $W_i$  cannot exceed the capacity:

$$W_i \leq A_i \quad (6-41)$$

The cost of artificial aerators includes the equipment itself, its installation, and finally, its operation and maintenance. The annual cost of the equipment and installation is a function of its horsepower capacity  $A_i$  and is denoted by the function  $C_{Ei}^A(A_i)$ . This annual cost function must properly take into account the shorter life span of aeration units compared to most other quality management alternatives if they too are considered simultaneously with aeration devices in the quality model.

A major portion of the annual operation and maintenance cost is for power. Assuming an operation of  $D_y$  time units a year and a power cost of  $P$  dollars per unit of power input, the annual operation costs are simply

$$C_{O_i}^A = D_y P W_i \quad (6-42)$$

Finally, annual labor and maintenance costs are often expressed as functions of the capacity  $A_i$ , or  $C_{Li}^A(A_i)$ . Thus, the total annual aeration cost at each site  $i$  can be written

$$C_i^A = C_{Ei}^A(A_i) + C_{O_i}^A + C_{Li}^A(A_i) \quad (6-43)$$

These costs can be incorporated into the objective function (6-23).

The estimation of the minimum cost combination of capacities  $A_i$  and the power inputs  $W_i$  is simplified by assuming that the oxygen transfer per unit of time  $H_i$  represents an average value throughout that unit of time when aerators are operating at a constant power input  $W_i$ . Since there is no need for (and higher costs associated with) extra capacity, power input per unit of time can be equated to capacity  $A_i$ . With this assumption and from Eq. (6-39),

$$OR_i = \zeta H_i A_i = \zeta H_s k_i DOD_i A_i \quad (6-44)$$

in which the dissolved oxygen deficit  $DOD_i$  and the capacity  $A_i$  are unknowns. The total annual cost is now a function of only the capacity  $C_i^A(A_i)$  at each site  $i$ .

#### 6-4.5 Waste-Water Transport Models

In addition to the alternatives just discussed, there is the option of transporting untreated or partially treated waste waters to other locations in or outside the region for further treatment and/or improved assimilation. Central or regional

treatment facilities may be attractive alternatives to local decentralized waste-water treatment, particularly in some metropolitan areas. The waste-water transport costs would be offset in part by lower marginal capital and operating costs obtained by economies of scale and increased operating efficiencies associated with increased treatment capacity. There also may be cost savings associated with releasing waste waters within or outside the region at locations having greater assimilative capacities, and benefits derived from increased water quality at sites where assimilative capacities are limited or where higher treatment efficiencies would otherwise be required.

Outlined in this subsection are some models similar to those proposed by Chi,<sup>20</sup> involving multiwaste sources and multiwaste treatment sites and waste-water discharge sites. The trade-offs are between waste-water treatment at the waste source and bypass piping for treatment and disposal at other sites. In addition, any of the previously discussed water-quality control alternatives can be considered simultaneously with bypass piping and treatment alternatives, if appropriate. In this subsection, however, only treatment and bypass piping are discussed.

The objective of water-quality management will be as before [Eq. (6-23)], namely, to find some politically effective combination of treatment and effluent charges plus capital and operating costs of waste-water diversion required to achieve specified waste-water effluent and/or stream-quality standards such as those expressed by Eqs. (6-24), (6-25), and (6-26). Waste-water diversion for further treatment and disposal is to be accomplished by piping and pumping.

Consider a metropolitan region defined by a set of discrete sites, each site denoted by the subscript  $i$ ,  $k$ , and  $l$ . The subscripts  $i$  designate existing or potential waste-water disposal sites. All other sites are subscripted by  $k$  or  $l$ . Letting  $F_{kl}$  be the waste-water flow from site  $k$  to site  $l$ , then the influent and effluent flow at each site  $l$  must equal

$$F_l = \sum_{\substack{k \\ k \neq l}} (F_{kl} + F_u) = \sum_k F_{kl} \quad (6-45)$$

where  $F_u$  is the flow of untreated waste water collected in the gravity sewers serving site  $l$ , if any.

The quantity of carbonaceous and nitrogenous BOD contained in the influent flow  $F_l$  at site  $l$  in any unit of time is the sum of the individual quantities in each flow  $F_{kl}$  plus the quantity of the untreated wastes  $BOD_{li}^C$  and  $BOD_{li}^N$  originating at site  $l$  during the corresponding unit of time:

$$BOD_l^C = \sum_{\substack{k \\ k \neq l}} (BOD_{kl}^C + BOD_{li}^C) \quad (6-46)$$

$$BOD_l^N = \sum_{\substack{k \\ k \neq l}} (BOD_{kl}^N + BOD_{li}^N) \quad (6-47)$$

The quantity of BOD in the effluent of the treatment facility at site  $l$  that is

transported to another site  $k$  will depend on the fraction of BOD removed,  $P_l$ , at site  $l$  and the fraction of the total flow  $F_l$  transported to site  $k$ :

$$\text{BOD}_{lk}^C = \frac{F_{lk}}{F_l} \text{BOD}_l^C (1 - P_l^C) \quad (6-48)$$

$$\text{BOD}_{lk}^N = \frac{F_{lk}}{F_l} \text{BOD}_l^N (1 - P_l^N) \quad (6-49)$$

where, as before,

$$\bar{P}_l^C \leq P_l^C \leq 1.0 \quad \text{and} \quad P_l^N = f_l(P_l^C) \quad (6-50)$$

Here  $\bar{P}_l^C$  is the existing treatment efficiency at site  $l$ , if any.

Clearly, all the flows from treatment sites  $l$  to each discharge site  $i$  define the total flow  $F_i$  released at each site  $i$ :

$$\sum_l F_{li} = F_i \quad (6-51)$$

Similarly, the total flow discharged into the natural receiving water or transported outside the region at all sites  $i$  must equal the total flow into the wastewater collection system at all entrance sites  $l$ :

$$\sum_i F_i = \sum_l F_l \quad (6-52)$$

Assuming that the indexing of sites is such that treatment is not provided at any disposal site  $i$ , then only the  $\text{BOD}_i^C$  and  $\text{BOD}_i^N$ , along with the wastewater flows  $F_i$ , are the unknown variables. If each released flow  $F_i$  does not significantly change the streamflow  $Q_{ij}$  between each waste discharge site  $i$  and quality site  $j$ , then there is no need to add each upstream waste-water flow to the natural flow at each site  $j$  when computing  $\text{BOD}_j$  and  $\text{DOC}_j$  by Eqs. (6-7), (6-8), and (6-12), respectively. If on the other hand the waste-water flow contributes significantly to the streamflow, this addition, and other adjustments discussed in the previous section on flow augmentation, will be necessary.

The minimum total cost of pumps and piping required to transport a given flow  $F_{kl}$  from one site  $k$  to another site  $l$  is an optimization problem in itself.<sup>20</sup> Without going into the details here, the minimum annual piping and pumping cost shall be denoted as  $C_{kl}^p(F_{kl})$ , a function of the quantity of flow from site  $k$  to site  $l$ . In this problem both the waste-water flows  $F_l$  and the fraction of BOD removed,  $P_l$ , at each site  $l$  are unknown decision variables. It is, therefore, necessary to define the cost of waste-water treatment as a function of the fraction of BOD removed  $P_l^C$  and  $P_l^N$  and the waste-water flow  $F_l$  at each site  $l$ . This annual cost shall be labeled  $C_l^T(F_l, P_l^C)$ . An objective might be to minimize the total weighted costs of waste-water treatment, transportation, and taxes.

$$\text{Minimize } \sum_l W_l \left[ C_l^T(F_l, P_l^C) + \sum_k C_{kl}^p(F_{kl}) \right] + \sum_i W_i T_i(\text{BOD}_i^C, \text{BOD}_i^N) \quad (6-53)$$

subject to the effluent and water-quality standards previously discussed.

Although the model as formulated permits flow in any direction defined, i.e., from site  $k$  to site  $l$ , or vice-versa, the solution (depending on the objective function) will usually indicate that the optimal operating policy involves flows in only one direction.

Multiperiod models would require that the capacity  $K_{kl}^p$  of the pipes and pumps be defined for all sites  $k$  and  $l$  and periods  $t$ :

$$K_{kl}^p \geq F_{klt} \quad (6-54)$$

$$K_{lk}^p \geq F_{lkt} \quad (6-55)$$

The annual capital and installation piping and pumping cost would be defined as a function of the capacity  $C_{kl}^p(K_{kl}^p)$  rather than flow, and the operating costs would be defined for each period  $t$ . In either the single or multiperiod case, common sense will often permit the elimination of many obviously nonoptimal alternatives, thereby simplifying the model and making it more amenable to efficient solution techniques.

Because of economies in scale on both treatment-plant construction and in pipe and pump installation, this problem should be viewed as a capacity expansion problem. One approach for estimating the capacity and timing of future system expansions would be to estimate the waste-water flows and BOD loads at future periods of time, solve static models for those various future periods to narrow down the number of alternatives, and then employ a discrete dynamic programming approach for estimating optimal capacity expansion schedules.<sup>21</sup>

One other problem needs mentioning—that of the desired redundancy of system components that are not gravity fed. Some redundancy is often needed in order to permit temporary reductions in system capacities for maintenance and repair. Any proposed system of treatment facilities, pipes, and pumps should be of sufficient capacity to permit reasonable operation during times when portions of the system are inoperable. This redundancy, of course, adds to the total cost of the treatment-bypass piping system.

## 6-5 LAKE AND ESTUARINE QUALITY-CONTROL MODELS

### 6-5.1 Lake-Quality Models

The models discussed above are more appropriate for streams and rivers than for lakes and estuaries. Lakes and estuaries, however, warrant some separate discussion because of the difficulties in applying some of the previously mentioned models to them. The state-of-the-art in modeling lakes is relatively young in comparison to that of modeling streams and estuaries.

Because of the variations in lake temperatures, wind and wave action, thermocline depth, and the like, the quality of fresh-water lakes and their entire ecosystems can change markedly and randomly. Only the most preliminary



mathematical models of lakes have been investigated. Much more research is needed, especially using stochastic models, before reliable parameters can be defined in order to derive waste-transfer coefficients and to predict the effectiveness of various schemes for controlling lake quality and the processes of eutrophication. These control schemes would include reduction of waste and nutrient inputs, artificial reaeration and mixing, dredging or flushing of bottom deposits, and lake-water discharge through multilevel outlets.

During critical lake-quality periods, approximations of stream conditions have been made to predict the quality near the lake surface. The depth of the lake can be assumed to be equal to only the depth of the epilimnion, i.e., the distance from the lake surface to the thermocline. Assuming a vertically and laterally mixed epilimnion of constant volume, the average time of flow through the epilimnion is simply its volume divided by the surface discharge from the lake. A rough estimate of the average BOD and dissolved oxygen concentration, DOC, in lakes can be made by setting the distance-divided-by-velocity term in the integrated form of Eqs. (6-4) and (6-9) equal to the average time of flow. This enables one to compute the BOD and dissolved oxygen transfer coefficients  $b_H$  and  $d_H$  within and upstream of the point of lake discharge. The DOC in the discharged flow is often considerably different than that in the flow just prior to discharge due to aeration during discharge. Obviously, if the lake release is from the hypolimnion, then both the DOC and BOD of the released water cannot be predicted from knowing only the quality of the epilimnion.

For some lakes, especially the smaller ones, it may be reasonable to consider the epilimnion completely mixed, longitudinally as well as vertically and laterally. The change in the mass of any waste or nutrient equals the mass input less the mass output less the mass decay, if any. The decay may result from biological, chemical, or physical processes in the epilimnion and/or the transfer of mass to or from the hypolimnion. Assuming a constant nutrient or waste input of  $N$  ( $MT^{-1}$ ), which has a net decay rate constant of  $K$  ( $T^{-1}$ ), a mass balance can be written for any interval of time  $\Delta T$ . Letting  $V$  equal the constant volume of the epilimnion ( $L^3$ ),  $Q$  the constant discharge flow ( $L^3T^{-1}$ ), and  $C_N$  the concentration of the material  $N$  ( $ML^{-3}$ ),

$$V(\Delta C_N) = N(\Delta T) - QC_N(\Delta T) - KC_NV(\Delta T) \quad (6-56)$$

Dividing through by  $V(\Delta T)$  and letting  $\Delta T \rightarrow 0$ ,

$$\frac{dC_N}{dt} = \frac{N}{V} - \frac{QC_N}{V} - KC_N \quad (6-57)$$

which when integrated yields a predictive equation for the concentration  $C_N(t)$  at time  $t$ . Assuming  $C_N(0) = 0$ ,

$$C_N(t) = \frac{N}{Q + KV} (1 - e^{-t(Q/V + K)}) \quad (6-58)$$

The equilibrium concentration  $C_{Ne}$  can be determined by setting  $t = \infty$  in Eq. (6-58) or by letting  $dC_N/dt = 0$  in Eq. (6-57):

$$C_{Ne} = \frac{N}{Q + KV} \quad (6-59)$$

Solving for the time  $t_\alpha$  required to reach a given fraction  $\alpha$  of the equilibrium concentration, i.e.,  $C_N(t)/C_{Ne} = \alpha$ , results in

$$t_\alpha = \frac{-V}{Q + KV} \ln(1 - \alpha) \quad (6-60)$$

With  $K$  equal to 0.25, lakes the size of Lake Michigan, if perfectly mixed (and they are not), would only take approximately 20 days to reach a concentration within 90 percent of its equilibrium concentration. A perfectly conservative substance ( $K = 0$ ) would take considerably longer.

Similar equations can be developed to estimate the concentrations and times associated with a decrease in a pollutant concentration. For the perfectly mixed lake epilimnion having an initial waste nutrient concentration  $C_0$  (at time  $t = 0$ ), possibly from a spill, and a constant volume  $V$ , discharge  $Q$ , and net decay coefficient  $K$ , the change in concentration with respect to time, as seen from Eq. (6-57), equals

$$\frac{dC}{dt} = -C\left(\frac{Q}{V} + K\right) \quad (6-61)$$

Thus the concentration  $C(t)$  at any time  $t$  is

$$C(t) = C_0 e^{-t(Q/V + K)} \quad (6-62)$$

Again, one can solve for the time  $t_\alpha$  required for the epilimnion to reach a fraction  $1 - \alpha$  of the initial concentration  $C_0$ , i.e.,  $C(t)/C_0 = 1 - \alpha$ :

$$t_\alpha = \frac{-V}{Q + KV} \ln(1 - \alpha) \quad (6-63)$$

This is precisely the same as Eq. (6-60). Additional methods of estimating pollutant removal times due to natural process in stratified lakes can be found in a paper by Sweers.<sup>22</sup>

Equations such as (6-59) can be used to estimate values of the input mass rates  $N$  that will result in average equilibrium concentrations  $C_{Ne}$  no greater than the maximum quantities defined by standards. Less-than-perfect mixing can be assumed by varying the flow and volume parameters.

The equilibrium dissolved oxygen concentration in a perfectly mixed epilimnion can also be estimated. The differential equation equating the time rate of change in dissolved oxygen concentration to a function of the  $BOD_i$  concentration from each source  $i$ , the DOC and DOD concentrations, the

deoxygenation rate constant  $K_{1i}$ , the volume  $V$  and flow  $Q$ , and the mass rate of dissolved oxygen input, OR, can be written:

$$\frac{d(\text{DOC})}{dt} = \frac{(\text{OR}) - Q(\text{DOC})}{V} - \sum_i [(K_{1i}^C) \text{BOD}_i^C + (K_{1i}^N) \text{BOD}_i^N] + K_2(\text{DOD}) \quad (6-64)$$

Before the equilibrium dissolved oxygen concentration  $\text{DOC}_e$  can be estimated, it is necessary to compute the equilibrium BOD concentrations  $\text{BOD}_{ei}$ . Using Eq. (6-59) and denoting, as before, the variable  $P_i$  as the fraction of BOD removed, the equilibrium BOD concentration is

$$\text{BOD}_{ei} = \frac{\text{BOD}_i(1 - P_i)}{Q + V(K_{1i} + K_3)} \quad (6-65)$$

where as before,  $K_3$  is the net sedimentation rate constant. By setting  $d(\text{DOC})/dt = 0$ , in Eq. (6-64), the equilibrium dissolved oxygen concentration equals

$$\text{DOC}_e = \frac{\text{OR} - V \left\{ \sum_i [(K_{1i}^C) \text{BOD}_{ei}^C + (K_{1i}^N) \text{BOD}_{ei}^N] \right\} + K_2 V \text{DOS}}{Q + K_2 V} \quad (6-66)$$

The mass rate of dissolved oxygen input OR is equal to the flow  $Q$  times the dissolved oxygen concentration in the inflow. Clearly, if the mass rate of dissolved oxygen input OR equals  $Q \cdot \text{DOS}$  and there is no BOD, then the equilibrium concentration will equal the saturation concentration.

It should be emphasized again that these models are extremely simplified for most lake-quality problems. (See, for example, Bella<sup>23</sup> and Chen and Orlob,<sup>24</sup> who have structured simulation models for predicting the dissolved oxygen concentration and other quality parameters as a function of time and lake depth.) Nevertheless, with some judgment as to appropriate values of  $Q$  and  $V$ , they can be used as a very preliminary means of estimating average pollutant concentrations in the upper layers of lakes and in predicting the average performance of various quality control alternatives that have already been discussed. Equations for predicting intermediate and equilibrium concentrations resulting from various types of time-varying pollutant flows into completely mixed lakes are summarized by O'Connor and Mueller.<sup>25</sup> They use these equations to predict the chloride concentrations in the Great Lakes.

While average pollutant concentrations in mixed lakes or lake epilimnions may be of interest for preliminary water-quality management planning, localized peak concentrations can cause fundamental changes in the ecology of entire lake systems. Rodin<sup>26</sup> has proposed some models for examining localized peak concentrations. His models are somewhat more complex than those discussed above and require additional data and assumptions.

### 6-5.2 Estuary-Quality Models

Mathematical modeling of salt-water estuaries is considerably more advanced than that of fresh-water lakes. This is because the mixing and transport of materials due to tides dominates the other processes such as wind action and thermal stratification, and tidal phenomena can be relatively easily observed and predicted. Therefore, it appears that deterministic models have greater utility in estuaries than in rivers or lakes.

This subsection will discuss some alternatives to the integration of Eq. (6-2) for estimating material (BOD, DOC, chlorides, etc.) transfer coefficients, e.g., the  $b_{ij}$  and  $d_{ij}$  of Eqs. (6-7) and (6-10). To do this, some assumptions are needed which differ from those used above.

For the purposes of the model that follows, the estuary is divided into  $n$  reaches, each reach denoted by the subscript  $i$ . The reaches are defined so that the parameters that affect the change in dissolved oxygen concentration within that reach are constant. Waste discharges from each source within a specific reach are assumed to have similar characteristics, even though their locations differ. Similarly, the effect of a given waste discharge into a specific reach is to be uniform throughout that reach. Therefore, the lengths and locations of these reaches must be chosen to assure that such approximations and assumptions will result in a reasonably accurate portrayal of the actual estuarine waste, dissolved oxygen, and other material concentrations.

To predict both the BOD and dissolved oxygen concentrations, DOC, in reaches of the estuary,<sup>27</sup> two-stage models can be developed. The first stage consists of one equation for each reach of the estuary. From Eq. (6-2), a mass balance of the BOD concentrations (BOD<sup>C</sup> or BOD<sup>N</sup>) in any reach requires that the rate of change of the BOD in reach  $i$ ,  $V_i (dBOD_i/dt)$ , equals the net amount of BOD transported by the flow  $Q_{i-1,i}$  from reach  $i-1$  to reach  $i$  and the flow  $Q_{i+1,i}$  from reach  $i+1$  to  $i$ ,

$$Q_{i-1,i}[zBOD_{i-1} + (1-z)BOD_i] + Q_{i+1,i}[zBOD_{i+1} + (1-z)BOD_i]$$

plus the net amount of BOD transported by diffusion,

$$E_{i-1,i}(BOD_{i-1} - BOD_i) + E_{i+1,i}(BOD_{i+1} - BOD_i)$$

less the BOD that is decayed or that settles to the bottom,

$$V_i(K_{1i} + K_{3i})BOD_i$$

plus the BOD load imposed on the reach by runoff, scour, and all the controlled discharges into the reach,  $BR_i$ . Thus,

$$V_i \frac{dBOD_i}{dt} = Q_{i-1,i}[zBOD_{i-1} + (1-z)BOD_i] \\ + Q_{i+1,i}[zBOD_{i+1} + (1-z)BOD_i]$$

$$\begin{aligned}
 &+ E_{i-1,i}(BOD_{i-1} - BOD_i) \\
 &+ E_{i+1,i}(BOD_{i+1} - BOD_i) \\
 &- V_i(K_{1i} + K_{3i})BOD_i + BR_i
 \end{aligned} \tag{6-67}$$

where  $V_i$  is the volume ( $L^3$ ) in reach  $i$ ,  $z$  is the advection factor that could also be denoted as  $z_{ij}$  between each pair of reaches  $i$  and  $j$ , and  $E_{ij}$  is the eddy exchange coefficient from reach  $i$  to reach  $j$  ( $L^3T^{-1}$ ). The eddy exchange coefficient between two successive reaches is related to the dispersion coefficient  $D$ , the cross-sectional area  $A$ , and the length  $w$  of the adjacent reaches:

$$E_{ij} = \frac{D_{ij}A_{ij}}{0.5(w_i + w_j)} \tag{6-68}$$

The BOD variables are concentrations ( $ML^{-3}$ ), and the  $BR_i$  variable is a mass input rate ( $MT^{-1}$ ) that may be partially controlled.

Converting Eq. (6-67) to a steady-state one requires the assumption that, for the time interval involved, say, a complete tidal cycle,  $dBOD_i/dt = 0$  and  $BR_i$  is constant. Equation (6-67) for each of  $n$  reaches can then be written

$$B_{i,i-1}BOD_{i-1} + B_{ii}BOD_i + B_{i,i+1}BOD_{i+1} + BR_i = 0 \tag{6-69}$$

where

$$\begin{aligned}
 B_{i,i-1} &= zQ_{i-1,i} + E_{i-1,i} \\
 B_{ii} &= (1-z)(Q_{i-1,i} + Q_{i+1,i}) - E_{i-1,i} - E_{i+1,i} - (K_{1i} + K_{3i})V_i \\
 B_{i,i+1} &= zQ_{i+1,i} + E_{i+1,i}
 \end{aligned}$$

Equation (6-69) represents a set of simultaneous equations, the number of unknowns,  $BOD_i$ , equaling the number of equations. In matrix notation,

$$B(BOD) - (BR) = 0 \quad \text{or} \quad (BOD) = -B^{-1}(BR) \tag{6-70}$$

where  $B$  is a square tridiagonal matrix ( $n \times n$ ), and  $BOD$  and  $BR$  are column vectors ( $n \times 1$ ). The elements in the matrix  $-B^{-1}$  are the coefficients  $b_{ji}$ , the BOD concentration ( $ML^{-3}$ ) in reach  $j$  resulting from a unit BOD input ( $MT^{-1}$ ) in reach  $i$ .

To solve for the steady-state dissolved oxygen concentration  $DOC_i$  in each reach  $i$ , an equation similar to Eq. (6-67) can be derived:

$$\begin{aligned}
 V_i \frac{dDOC_i}{dt} = 0 &= Q_{i-1,i}(zDOC_{i-1} + (1-z)DOC_i) \\
 &+ Q_{i+1,i}(zDOC_{i+1} + (1-z)DOC_i) \\
 &+ E_{i-1,i}(DOC_{i-1} - DOC_i) \\
 &+ E_{i+1,i}(DOC_{i+1} - DOC_i) \\
 &- V_i K_{1i}^C BOD_i^C - V_i K_{1i}^N BOD_i^N \\
 &+ V_i K_{2i}(DOD_i) + OR_i
 \end{aligned}$$

where each term is as previously defined. The variable OR is expressed as a mass rate ( $MT^{-1}$ ), whereas the dissolved oxygen and BOD variables are concentrations ( $ML^{-3}$ ). For each reach  $i$ , Eq. (6-71) is equivalent to

$$A_{i,i-1}DOC_{i-1} + A_{ii}DOC_i + A_{i,i+1}DOC_{i+1} - V_i K_{1i}^C BOD_i^C - V_i K_{1i}^N BOD_i^N + V_i K_{2i} (DOS)_i + OR_i = 0 \quad (6-72)$$

$$\begin{aligned} \text{where } A_{i,i-1} &= zQ_{i-1,i} + E_{i-1,i} \\ A_{ii} &= (1-z)(Q_{i-1,i} + Q_{i+1,i}) - E_{i-1,i} - E_{i+1,i} - V_i K_{2i} \\ A_{i,i+1} &= zQ_{i+1,i} + E_{i+1,i} \end{aligned}$$

In matrix form, combined Eqs. (6-70) and (6-72) can be written

$$A(DOC) + VK_1^C B_C^{-1} (BR^C) + VK_1^N B_N^{-1} (BR^N) + VK_2(DOS) + OR = 0 \quad (6-73)$$

where  $A$ ,  $B_C$ , and  $B_N$  are  $(n \times n)$  matrices;  $V$ ,  $BR^C$ ,  $BR^N$ ,  $DOC$ ,  $OR$ , and  $DOS$  are  $(n \times 1)$  column vectors; and  $K_1^C$ ,  $K_1^N$ , and  $K_2$  are  $(1 \times n)$  row vectors. Equation (6-73) can be rewritten

$$DOC = -VK_1^C A^{-1} B_C^{-1} (BR^C) - VK_1^N A^{-1} B_N^{-1} (BR^N) - A^{-1} VK_2(DOS) - A^{-1}(OR) \quad (6-74)$$

which is equivalent to

$$DOC_j = \sum_i (d_{ji}^C BR_i^C + d_{ji}^N BR_i^N + \delta_{ji}(OR)_i) - \gamma_j \quad (6-75)$$

where  $d_{ji}$  = the increase in dissolved oxygen concentration in reach  $j$  due to a unit decrease in the concentration of BOD in reach  $i$ . The matrix of  $d_{ji}$  equals  $D = -VK_1[BA]^{-1}$

$\delta_{ji}$  = the increase in dissolved oxygen concentration in reach  $j$  due to a unit increase in the mass rate of oxygen in reach  $i$ . The matrix of

$$\delta_{ii} = -A^{-1}$$

$\gamma_j$  = an element in the vector  $A^{-1}VK_2(DOS)$

The influent BOD concentrations  $BR_i$  in each reach  $i$  include the controlled wastes discharged into reach  $i$ . If  $BOD_{ki}(1 - P_k)$  is the BOD mass rate ( $MT^{-1}$ ) entering reach  $i$  from site  $k$  along that reach after the removal of a fraction  $P_k$  of total waste  $BOD_{ki}$ , and  $BR'_i$  is the mass rate of uncontrolled BOD input from scour and runoff, then the influent BOD concentration  $BR_i$  for each reach  $i$  equals

$$BR_i = \frac{1}{Q_i} \left\{ \sum_k [BOD_{ki}(1 - P_k)] + BR'_i \right\} \quad (6-76)$$

If desired, Eqs. (6-75) and (6-76) can be used to replace Eqs. (6-10), (6-11), and (6-12) for estuarine systems.

## 6-6 DISCUSSION AND REVIEW

This chapter has outlined only a few models and techniques that can be used to define and evaluate various water pollution control alternatives. A complete report on the state-of-the-art of the application of systems approaches and techniques to the formulation and analysis of water-quality problems would require much more space than allowed here.

Rather than attempting to be a comprehensive state-of-the-art report on water pollution control models, this chapter merely reviews some analytical and computational methods of analyzing dissolved oxygen management systems, from waste-water collection, treatment, and disposal to its subsequent effect on receiving water bodies. Emphasis has been placed on problem definition and model development and not on model-solution techniques. Also emphasized has been the difficulty of defining and quantifying environmental quality and similar public policy objectives. This often leads to differences between the optimal solutions to water-quality management models and the eventual solution as chosen by the political process. Not understanding this limited meaning of the word optimization has tended to confuse some who have tried to read and understand the literature on water-quality models.

The models that have been presented are not intended to be definitive; they only illustrate how various water-quality-control subsystems can be modeled and analyzed. While various groups of alternatives were discussed and modeled separately, any of the subsystem models can be combined into a more comprehensive model should conditions dictate. Problem definition, modeling, and analysis involves considerable judgment in addition to some mathematical and computational skills. Just how this is accomplished in any particular situation depends not only on the problem itself but also on the skill of the systems analyst(s) and the available data, programming algorithms, and computational facilities. Clearly, the systems models and techniques illustrated in this chapter are not substitutes for judgment—they are only a means of enhancing this judgment by providing information and opportunities that otherwise might not be available.

Experience to date has demonstrated that some of the more obvious types of information and opportunities provided by systems approaches to water-quality problems include: (1) an increased capability for defining and evaluating possible alternatives and of keeping a wider range of options open and available for analysis at each level of decision-making; (2) an improved capacity for testing assumptions and data to estimate the effects of economic, hydrologic, political, and technological uncertainties; (3) a method whereby all assumptions and judgments and the consequences of these assumptions and judgments are made explicit and available for all to see and question should they desire to do so (to some this feature seems to be a distinct disadvantage); and (4) a means of communication between all the participants in the planning and evaluation stages.

The analysis of water pollution control problems, like any public policy planning process in a dynamic environment, is never completed. It is an iterative process—one that involves continual updating of information and techniques of analysis. Since this activity costs money, the extent of any increase in the types and precision of the data collected or increase in modeling sophistication depends on the additional benefits achieved. These costs and benefits are often difficult to assess, especially when environmental quality problems are involved. Needless to say, however, if the systems analysis studies that are undertaken are done well at whatever level of decision-making, the increased data base should enhance the content of the debate over what decision to make—a debate that can center on which assumptions are better than others rather than on what the best solution is given a particular set of assumptions.

Often questioned is the effectiveness of systems approaches for solving current and future water pollution and other environmental quality control problems. The answer is simple; systems analysis is not a particularly effective technique for problem solving but rather an approach for problem analysis. It consists of procedures that can be used to find potential solutions to problems, but not necessarily optimal ones. The principal reason for this is that it is practically impossible for analysts or the policy makers to define or quantify the optimal policy objective. Nevertheless, even without precise objective functions, systems techniques can be used to define sets of politically and economically effective alternatives. Thus, the relevant question is not how many optimal solutions of water-quality models have been implemented but rather how beneficial the results of such solutions and analyses have been to the decision-making process. One can only observe and speculate that the combined use of management and simulation modeling for defining and evaluating water-quality control alternatives has and will continue to assist those responsible for water-quality management planning and policy. Much of the current literature supports this conclusion.

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