5

SURFACE-WATER QUANTITY MANAGEMENT MODELS

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5-1 INTRODUCTION

By far the majority of the benefits and damages resulting from the world's supply of water stem from the use or misuse of surface water and its related land resources. Surface water provides the major portion of the world's agricultural, industrial, and domestic water supplies. It is the sole source of water for hydroelectric energy production, for water-based recreation, and for the maintenance of fish and other aquatic animal and plant life. Finally, together with various land-use policies, surface water is the major cause of flood damage. Perhaps for these reasons the first efforts in the use of graphical and mathematical analysis techniques for water-resources planning were directed toward the regulation of surface-water flows. Today there are literally hundreds of procedures and models proposed and used for analyzing surface-water quantity problems.

While the regulation of surface water is an important component of the total water-resources management problem, comprehensive analyses of regional water-resource management alternatives must consider the conjunctive use of ground and surface waters and the control of water quality as well as quantity. This chapter will focus on only that aspect of the management problem concerned with the definition and evaluation of alternatives for controlling the allocation and distribution of surface-water flows.

Without attempting to review each of the procedures currently used for analyzing surface-water management alternatives, the models presented in this chapter will, it is hoped, capture the essence of most of these techniques, and at the same time provide some additional information on the risk associated with various investment and operating policies for surface-water development and management.

Specifically, the models in this chapter will serve as a means of illustrating how one can make preliminary estimates of (1) the desired amount and reliability of various yields or allocations of water to each consumptive or nonconsumptive use, (2) the requirements, if any, for over-year as well as within-year reservoir storage capacity based on the sequence of historical or simulated flows, (3) the requirements, if any, for structural flood-control measures and the desired level of flood protection at each potential damage site, and (4) the reservoir operating rule curves, i.e., the identification of the flood storage-capacity requirements and the various zones within the active storage capacity of each reservoir from which withdrawals or releases can be made to meet specific downstream water demands.

A number of intentional approximations will be made to render the models compatible with currently and readily available computer solution techniques (software) and capacities (hardware). The emphasis here will be on models that can be readily applied by water-resources planners even in situations where there exist limited data and limited computer capacity.

Before developing these preliminary screening models, it seems instructive to review the various types of models that are often used for surface-water analyses. Following this review, a few of the functions required for the estimation of the annual net benefits associated with various investment and operating policies will be defined for use throughout the chapter. Some procedures for at least partially overcoming the difficulties in quantifying nonmonetary benefits will also be discussed. The remainder of the chapter will be devoted to model development and analysis.

5-2 MODEL TYPES

Numerous types of models have been developed for the analysis of surface-water-resource systems. Models can be classified as either optimization or simulation models, static or dynamic models, deterministic or stochastic (probabilistic) models, and investment and operating policy or strictly operating policy models. All model types have their strengths as well as their limitations. There is no one optimal model type for all water-management problems. The problem is not one of identifying which model type is better than another but rather one of determining the desired trade-offs among the kinds of information each model type can best provide, given limitations of time, money, and computer capacity.

Each type of model has its place in river basin planning. However, knowledge of the advantages as well as the limitations of many model types must be understood before these quantitative techniques can be used effectively as tools complementing the more traditional quantitative and qualitative planning methods.

5-2.1 Optimization and Simulation Models

Perhaps the most frequently used surface-water models are those that consist simply of a sequence of mathematical and logical statements describing the design and operation of a river basin system. Such a sequence of statements written for any particular river basin, together with a series of historical or synthetic streamflows at various gaging stations in that basin, provides a means of simulating the operation of that system in order to predict and analyze its performance. Simulation models developed for solution on digital computers have proven to be very effective tools for estimating the future hydrological and economical performance of any proposed surface-water-management policy.[1]

While simulation models are effective methods for evaluating alternative configurations of reservoir and hydroelectric power plant capacities, water-use allocation targets, operating policies, and the like, they are not a very effective means for choosing or defining the best combination of capacities, targets, and policies. For this purpose, optimization models have proven to be effective, if not for finding the best solution, at least for eliminating from further consideration the worst solutions.[18,19]

There are several reasons why optimization models cannot determine the exact optimal solution to water-resource-management problems. One major limitation is mathematical. Solution techniques, called *algorithms*, for optimization models often require some simplifying assumptions that may or may not be limiting. Nonlinear cost, benefit and loss functions, and nonlinear expressions required for defining evaporation losses, hydroelectric energy production, and some combinations of flood-control alternatives are often approximated by piecewise linear functions. These assumptions and approximations may or may not result in an efficient and effective preliminary screening of investment and operating policy alternatives.

A second limitation of optimization models is computational. The modeling of even relatively small surface-water systems may result in more mathematical expressions, constraints, and variables than can reasonably and accurately be simultaneously considered on existing computer facilities. Hence, again some simplifying assumptions are often made to reduce the size of the models and the cost of their solution.

A third and perhaps the most limiting aspect of optimization models is the conceptual difficulty associated with the quantification and specification of a criterion for evaluating each possible management alternative. Public policy objectives are a mixture of monetary and nonmonetary goals, making quantification difficult. Furthermore, agreement among decision makers on what these goals should be, let alone the extent to which they are to be satisfied, is rarely achieved during the preliminary analysis phase of water-resource planning.

There are other important limitations related to the quantification of hydrologic, technologic and economic uncertainties, inaccurate and incomplete data, and the like.

All these mathematical, computational, conceptual, and data limitations restrict the use of optimization models to preliminary screening. Those alternative investment and operating policies that survive the preliminary screening process should be further analyzed, evaluated, and improved, using simulation or other techniques where appropriate. While simulation models share many of the same conceptual and data limitations, they are far less restrictive mathematically and computationally; hence, they are usually better suited for evaluating more precisely the alternatives defined by the optimization or preliminary screening models.

5-2.2 Static and Dynamic Models

Both optimization and simulation models can be further classified based on the assumed changes, if any, in the economic and/or hydrologic conditions in each successive year. Some surface models assume that the hydrometeorological processes that result in steamflows throughout the year do not vary significantly from year to year. In other words, while the actual or predicted streamflows may vary randomly from month to month and from year to year, the probability distributions of the streamflows at any particular time of the year are assumed to remain constant or static from one year to the next. In some areas this assumption may be questionable, especially if the annual streamflows are cyclic or if urbanization significantly alters the runoff characteristics of the watershed.

Models that are static with respect to hydrology may be either static or dynamic with respect to economics. Surface-water models which are static in an economic sense are structured to define and evaluate various investment and operating policies for a given economic condition forecast to exist some specified year in the future, say 2000 or 2020.

Dynamic economic models assume varying economic conditions over time and attempt to provide information relative to project staging and sequencing. Because the benefits derived from each water-resources project are hydrologically dependent on the immediate if not all upstream projects and operating policies, a dynamic analysis for the determination of project scheduling and staging must usually be made for the river basin as a whole.

The size of any reasonably detailed dynamic optimization model of an entire river basin often precludes its solution even on the most modern computer facilities. Although conceptually possible, it is usually computationally impractical to combine within a single model both the detail of a static analysis and the number of periods included in a dynamic analysis. In these cases, some snapshots of good investment and operating policy alternatives obtained from economically static models for various future years can be used to reduce the number of alternatives and the detail that would otherwise have to be included in dynamic economic analyses.[19]

5-2.3 Deterministic and Stochastic Models

Surface-water models can also be classified on the basis of whether or not future streamflows are assumed to be known. A model is hydrologically deterministic if the future unregulated streamflows are assumed to be known and are specifically defined in the constraint set of the model. If only the probability distributions of the unregulated streamflows are assumed to be known, the model is stochastic or probabilistic. The solutions to hydrologic stochastic models are usually conditional or dependent on possible future streamflows, reservoir levels, and other random variables.[20,21]

Stochastic optimization models often contain more variables and constraints than do deterministic models of the same system. This increase in the number of variables and constraints, together with limitations in computer capacity and speed, usually restrict the stochastic models to relatively small sub-basin problems. Nevertheless, information derived from stochastic analyses is often more useful for preliminary screening than is the information resulting from deterministic analyses. In this chapter, the screening models will be generally deterministic, but an analysis of the streamflows used in the models will permit some estimates of the probabilities associated with various regulated streamflows and water-use allocations. The procedures to be developed will combine some of the advantages of both deterministic and stochastic models in an effort to achieve a useful and practicable means for the preliminary definition and evaluation of investment and operating alternatives and their associated risks.

5-2.4 Investment and Operating Models

The final model classification to be discussed has to do with what economists term long-run and short-run planning. Long-run decisions include possible changes in physical facilities such as reservoir and hydroelectric power plant capacities, recreational structures, irrigation acreages and crops, and the like. These investment alternatives are among the problem variables for which optimal values are desired. What must be determined is the level of investment in each project as well as the policy for the coordinated operation of all projects.[19]

Short-run analyses, on the other hand, examine only the operating policy variables of a surface-water system. Project capacities and target outputs are assumed fixed or known. This permits the development of more detailed models for analyzing the operation of surface-water systems. Such systems might consist of several reservoirs of known capacity whose random inflows are serially and cross correlated.[21,22]

5-3 BENEFIT-LOSS AND COST CRITERIA

An investment model of any surface-water system consists of a set of mathematical expressions or constraints defining various possible investment and operating policy alternatives and a criterion or objective function used to compare or evaluate each of the possible alternatives. An ideal criterion would be a function relating the decision variables to the social welfare of one or more identifiable groups of individuals. Social welfare functions include numerous parameters, many of which are not easily expressed in monetary terms. Because these functions are difficult to quantify, a substitute means often used to define and evaluate sets of alternatives involves a combination of politically weighted economic efficiency objectives and a series of constraints for defining maximum and minimum permissible allocations or capacities.[11]

To define economic efficiency, three types of functions are required. These functions define the monetary benefits, losses, and costs associated with the values of various decision variables. While the total benefits and losses associated with many water uses cannot be expressed solely in monetary units, there are many that can be partially measured in these terms. These include hydropower production, irrigation, reservoir-based recreation, expected flood-damage reduction from reservoir capacity for flood storage and from channel improvement, and many industrial and municipal water allocations.[1] The benefits associated with a given allocation of water are usually defined as the lesser of either its opportunity cost, i.e., the least costly alternative means of achieving the same allocation from outside the system, or the willingness of consumers to pay for the allocation.

Figure 5-1 illustrates the concept of a planned or target allocation T and the typical relationship between a long-run benefit function $B(T)$ and a short-run benefit function $b(Q|T)$ which depends on the target T and the actual allocation Q. The long-run function reflects the benefits users receive when they have adjusted their plans in anticipation of receiving an allocation equal to the target T and actually receive it. The short-run function measures the benefits users receive when their actual allocation Q is less (e.g., Q_1) or more (e.g., Q_2) than their anticipated allocation T and they cannot completely adjust to the resulting deficit or surplus in the short run.

The short-run loss of any actual allocation Q equals the long-run benefit of the target allocation T minus the short-run benefit of the actual allocation Q, i.e., $B(T) - b(Q|T) = L(Q|T)$. Clearly when the actual allocation equals the target allocation, the short-run loss is zero. Hence, there is a different short-run benefit function for each value of the target allocation. However, the short-run losses associated with any deficit allocation $(T - Q_1)$ or surplus allocation $(Q_2 - T)$ may be relatively constant over a reasonable range of targets. Hence,

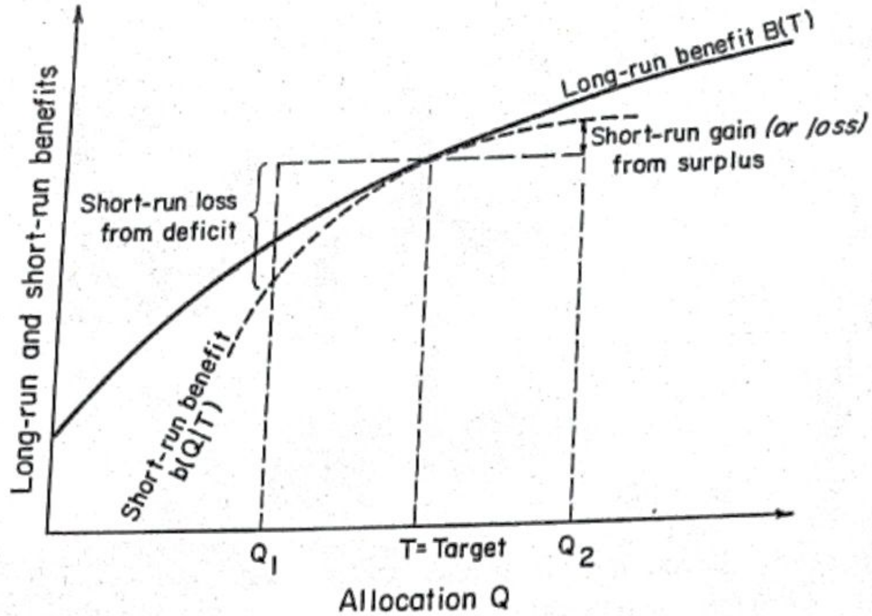


FIGURE 5-1
Long-run and short-run benefit functions.

it may not be necessary to define the loss as a function of the target T, but only of the deficit D or the surplus E, where D and E are defined by the constraint $Q = T - D + E$. (Additional detail on methods of defining these benefit-loss functions can be found in Maass et al.[10] and in chap. 9 of Dorfman et al.[11])

Annual cost functions incorporate both capital and operation, maintenance, and repair (OMR) costs. The annual capital costs are usually defined as functions of project capacities or allocation targets, whichever is appropriate. Given a total capital cost C', an annual discount rate (fraction) r, and a project life of n years, the equivalent uniform annual cost of the project equals the initial capital cost C' times the capital recovery factor, $[r(1+r)^n]/[(1+r)^n - 1]$.

Added to this equivalent annual cost of capital expenditures are the annual OMR costs. Annual operation, maintenance, and repair costs are often defined as either functions of targets or capacities or as functions of the capital costs. Costs that are functions of targets can be subtracted from the corresponding gross long-run benefits to derive long-run net benefit functions. This is not possible in cases where, for example, the benefits are functions of targets and the costs are functions of capacities.

The benefit, loss, and cost functions can be combined to define a measure of economic efficiency. Economic efficiency is clearly only one criterion for plan formulation and evaluation. Social welfare is, in part, dependent on the distribution as well as on the total amount of monetary benefits derived from any investment and operating policy. To consider alternative distributions of monetary benefits among various water users, the net benefits of each water user can be weighed relative to each other. A variety of weights can be assumed to generate a series of reasonable alternatives for further analysis and evaluation.

To summarize this section, it will be helpful to define some additional notation. Let each identifiable group of water users within a region be denoted by a particular value of the subscript i. Assume that their annual net monetary benefits B_i are weighted by the fraction W_i. The annual net monetary benefits each group i receives are the sum of the long-run annual benefits $B_{ij}(T_j)$ derived at various sites j from target allocations T_j, minus the sum of their short-run losses $L_{ij}(Q_j|T_j)$ and their annual costs $C_{ij}(K_j)$ for project capacities K_j. In addition the total costs that can be paid by each group i may be limited by their financial resources $\$_i^{max}$.

Given these definitions, the objective is to find those targets T_j, allocations Q_j, and capacities K_j at each site j that maximize the total annual weighted net benefits,

$$\text{Maximize} \qquad \sum_i W_i \cdot B_i \tag{5-1}$$

where the annual monetary net benefits to each group i equal

$$B_i = \sum_j [B_{ij}(T_j) - L_{ij}(Q_j|T_j) - C_{ij}(K_j)] \qquad \forall i \tag{5-2}$$

For noneconomic reasons, the range of allocations Q_j may be constrained:

$$Q_j^{min} \leq Q_j \leq Q_j^{max} \qquad \forall j \tag{5-3}$$

The above maximization may also be subject to budget constraints limiting the level of investment by various groups i,

$$\sum_j C_{ij}(K_j) \leq \$_i^{max} \qquad \forall_i \tag{5-4}$$

Finally, additional constraints may be desired, ensuring that no single group will incur a net loss:

$$B_i \geq 0 \qquad \forall i \tag{5-5}$$

Note that if the weighting fractions W_i are all equal to 1, the objective is simply that of maximizing economic efficiency subject to some limits on the allowable allocations and investment capital. In addition, if no benefits or losses are defined, i.e., $B_{ij} = L_{ij} = 0$, and constraint equations (5-5) are omitted, the problem is one of cost minimization. What was originally a benefit-cost analysis is now a cost-effectiveness analysis. Fixing the values of the targets T_j and capacities K_j changes the long-run analysis of both investment and operating policy alternatives to a short-run analysis of only operating policies.

As previously mentioned, national or regional income and its distribution are only two of many possible objectives. Others that are not so easily expressed in monetary terms might include environmental quality, social well-being, national security, regional growth and stability, and preservation of natural areas. Insofar as nonmonetary objectives can be quantified, they too can be explicitly incorporated into the objective function and/or the constraint set of models for defining and evaluating water-resource management alternatives. The results of these multiple-objective analyses will not yield the optimal set of weights or the optimal trade-offs among conflicting noncommensurable objectives; this is a political decision. However, such analyses can assist those responsible for making these political decisions by identifying the trade-offs among objectives that are feasible and efficient. (Additional information on multiple-objective analyses can be found in Chap. 10 of this text as well as chaps. 2, 3, and 9 in Dorfman et al.,[11] chaps. 21 and 22 in deNeufville and Marks,[12] and in Refs. 13 through 17.)

5-4 SURFACE-WATER STORAGE REQUIREMENTS

The primary purpose of surface-water reservoirs is to provide a means of regulating the distribution, both with respect to time and amount, of surface-water flows and volumes. The use of reservoirs for temporarily storing surface water often results in a net loss of total streamflow due to evaporation and seepage into the ground. While these possible hydrologic losses may not be desired, the social and economic benefits derived from the regulation of surface-water flows may offset any loss resulting from reduced annual flows and the costs of reservoir construction and operation. These increased benefits may stem from greater yields and reduced flood damages to downstream users, as well as from water-based recreational activities on the lakes created by the reservoirs. In addition, higher yields, together with increased storage heads, provide opportunities for increased hydroelectric power production and its associated benefits.

Whether or not a surface-water storage reservoir is desired at a particular site in a river basin depends, in part, on the economic net benefits that would be derived from the reservoir. In this section, mathematical expressions will be developed that define and describe the hydrologic and economic interrelationships that exist between reservoir capacity, yield, and operating policy. Later in the chapter these expressions will be included within a more comprehensive surface-water management model whose purpose is to estimate and evaluate the total system costs and benefits associated with numerous reservoir capacities, yields, and operating policies.

As illustrated in Fig. 5-2, reservoir storage-capacity requirements can be divided into three major components: (1) the active storage required for firm and secondary yields, (2) the dead storage desired for sediment collection, recreational development, and hydropower production, and (3) the flood storage capacity required to reduce downstream flood damages. The storage requirement for firm and secondary yields can be further subdivided into over-year and within-year requirements. Methods for deriving the functional relationships between storage and yields and between capacity and flood peak reduction will be discussed below. Once the functions defining the various

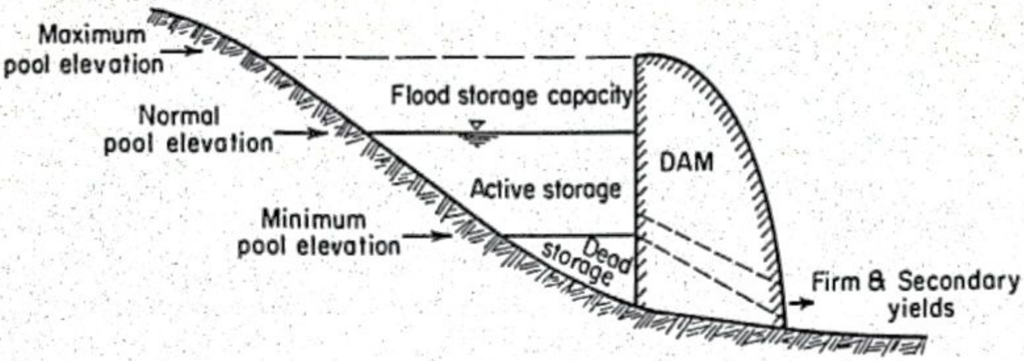


FIGURE 5-2
Reservoir storage zones.

storage requirements are developed, they can be used together with economic benefit, loss, and cost functions to estimate the desired total storage capacity and resulting net benefits at each potential reservoir site.

5-4.1 Over-Year Storage for Annual Yields

The analysis of the relation between yield and capacity is one of the most important aspects of reservoir planning. Most water users are concerned with what is termed a *safe* or *firm yield*. This yield is normally defined as the maximum quantity of water that can be "guaranteed" downstream from a particular reservoir site. This guaranteed firm yield is in part dependent on the active storage capacity, the distribution of inflows, and the reservoir operating policy.

Because flows into each reservoir are random, it is not possible to guarantee any yield with certainty. Firm yields have customarily been based on the historical streamflow record. Since in the future, flows less than the lowest flow of record may occur, some yields may be less than the historical firm yield. Yields greater than the firm yield are called *secondary yields*. Secondary yields may also be beneficial to certain users who depend primarily on firm yields. For example, secondary yields allocated for domestic water supplies may be used for lawn watering, car washing, and the like. Secondary yields at hydropower generating sites may be used for the production of surplus energy.

Each yield, whether firm or secondary, has associated with it a probability that it will be exceeded. These probabilities are usually estimated from the unregulated historical flows. These estimates can be improved if the historical flows are supplemented by synthetic flows having the same statistical characteristics. In any case, these probabilities are based on the assumption that there will be no marked changes in watershed characteristics which would significantly alter the distribution and amount of surface and subsurface runoff. If this assumption cannot be made, then rainfall-runoff models might be used to generate more precise unregulated streamflow data.

Various methods can be used to estimate the probability that any given flow will be exceeded. The method adopted in this chapter will be that commonly used by the U.S. Geological Survey to analyze flood and drought frequencies.[2] It involves the prediction of the mean number of random events that can occur in the future. The probability associated with such a number is called a *mean probability*.

The mean probability of any particular streamflow being equalled or exceeded is based on the assumption that any future flow has an equal probability of falling within any interval defined by a sequence of historical and/or synthetic streamflows. Suppose, for example, that there exists a record of n unregulated annual streamflows, hence $n + 1$ streamflow intervals. Arranging these flows in order and ranking them so that the largest streamflow has the lowest rank, $m = 1$, and the lowest streamflow has the highest rank, $m = n$, the mean probability that a future unregulated annual streamflow will equal or exceed a flow of rank m is $m/(n + 1)$. The expected recurrence interval of an annual flow of rank m is $(n + 1)/m$ years, the reciprocal of the mean probability.

Having an estimate of the mean probability of a given unregulated streamflow makes it possible to define the mean probability of any particular reservoir yield. To illustrate how this might be done, consider the following 9 year flow sequence. Each streamflow is in units of millions of cubic meters (10^6 m^3):

Year	1	2	3	4	5	6	7	8	9
Streamflow, 10^6 m^3	7	3	5	1	2	5	6	3	4
Rank, m	1	6	3	9	8	4	2	7	5

Without reservoir storage, the firm or safe yield of the above streamflow sequence is 1 million m^3/year. Its mean probability of being exceeded is $9/10 = 0.90$. The mean probability of a yield of 2 million m^3 being exceeded is 0.80. This also could be called a firm yield, although not as firm as a yield of 1×10^6 m^3. Thus with only 9 years of records, "firm" or "safe" means at most only 90 percent firm or safe. If this streamflow record were the critical portion of a much longer sequence, say, 99 annual flows, one could be 99 percent certain of the 1×10^6 m^3 firm yield. In other words, the meaning of firm yield is defined by the mean probability of that yield being exceeded, which in turn is a function of n, the total number of years of recorded streamflows. Once the firm yield is defined, all yields greater than the firm yield are secondary yields.

With reservoir storage, it is possible to increase the firm and secondary yields. For example, given the 9-year flow sequence defined above, if in year 4 (when the annual flow is 1×10^6 m^3) a stored volume of 1×10^6 m^3 were available, then this quantity could be released in order to increase the firm yield from 1 to 2×10^6 m^3. Additional increases in storage would permit further increases in firm yields, up to the mean annual flow (which for a firm yield having a mean probability of 0.9 is 4×10^6 m^3) less evaporation and other losses.

The storage required for various firm yields can be derived from conventional mass diagram analyses.[3] Figure 5-3 illustrates the mass curves associated with the 9-year streamflow sequence and the active storage require-

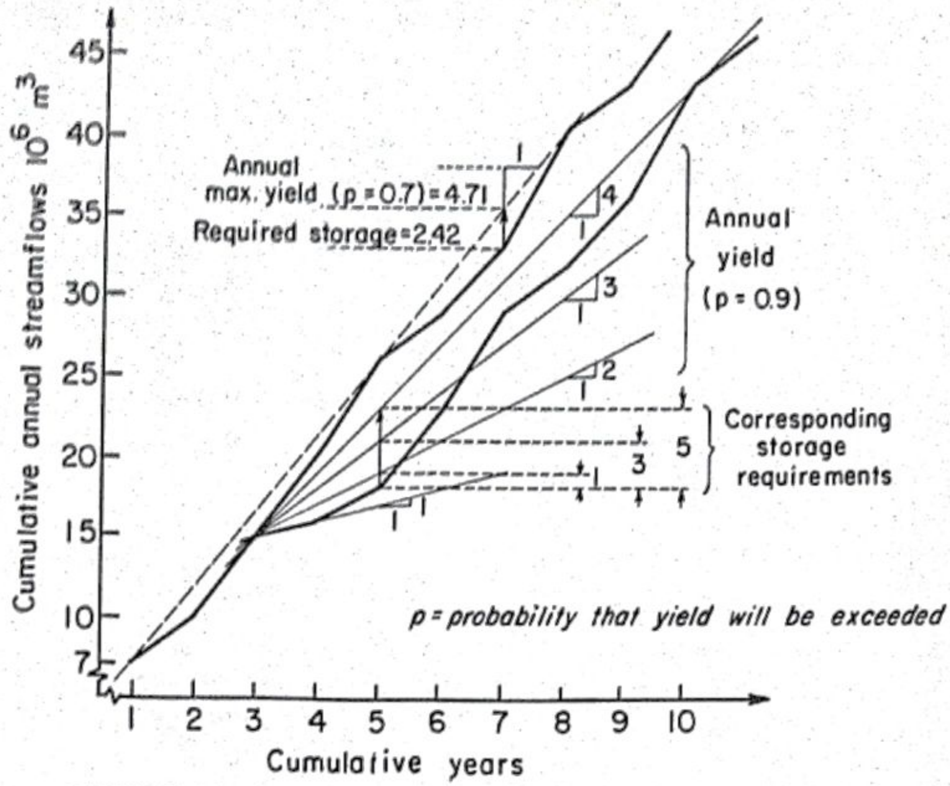


FIGURE 5-3
Mass diagram for 9-year streamflow sequence.

ments for firm yields of 1, 2, 3, and 4 × 10^6 m³. Each of these yields has a mean probability of 0.9 of being exceeded. Also shown in Fig. 5-3 is the storage required for the maximum firm yield having a mean probability of 0.7. This was derived by ignoring the two lowest flow years.

An alternative approach for estimating the storage requirements for various firm yields is to compute storage-yield curves. A convenient algorithm for use in calculating the required storage is a modified version of the sequent-peak algorithm.[4] Let S_y be the storage required at the beginning of year y, Y_p be the firm yield having a mean probability $p = n/(n+1)$ of being exceeded, and I_y be the streamflow in year y. Setting $S_0 = 0$, compute S_y, defined by Eq. (5-6), for up to twice the length of record, omitting those years having flows whose rank is greater than $p(n+1)$:

$$S_y = \begin{cases} Y_p - I_y + S_{y-1} & \text{if positive} \\ 0 & \text{otherwise} \end{cases} \tag{5-6}$$

The maximum of all S_y is the required storage for a yield of Y_p.

The upper half of Table 5-1 illustrates the use of this algorithm to compute the storage required for a firm yield ($p = 0.9$) of 3 × 10^6 m³. Using this procedure for computing the storage required for various firm yields results in the storage-yield functions illustrated in Fig. 5-4. Note the correspondence between the information derived from the mass diagram analysis and from the storage-yield function for those firm yields having mean probabilities of 0.9 and 0.7.

Associated with any firm yield less than the mean annual flow are various secondary yields. Assuming the firm yield to be the maximum yield having a mean probability of 0.9, Fig. 5-5 illustrates the maximum available secondary yields having mean probabilities of 0.7 and 0.5. Any combination of firm- and secondary-yield functions can be computed using an expanded version of Eq. (5-6):

$$S_y = \begin{cases} \sum_p \alpha_{py} Y_p - I_y + S_{y-1} & \text{if positive} \\ 0 & \text{otherwise} \end{cases} \tag{5-7}$$

Table 5-1 CALCULATION OF ANNUAL STORAGE-YIELD FUNCTIONS

Year y	Firm yield $\{Y_{0.9}$	Stream-flow $- I_y$	Previous storage $+ S_{y-1}\}^+$	Current storage $= S_y$
				0
1	3	7	0	0
2	3	3	0	0
3	3	5	0	0
4	3	1	0	2
5	3	2	2	3*
6	3	5	3	1
7	3	6	1	0
8	3	3	0	0
9	3	4	0	0

The second cycle is identical to first, since $S_9 = S_0$.
*Maximum required storage = 3 × 10⁶ m³.

Year y	Firm yield $\{Y_{0.9}$	Incremental secondary yield $+ Y_{0.7}$	Stream-flow $- I_y$	Previous storage $+ S_{y-1}\}^+$	Current storage $= S_y$
					0
1	3	1	7	0	1
2	3	1	3	0	0
3	3	1	5	1	2
4	3	—	1	0	3*
5	3	—	2	2	2
6	3	1	5	3	0
7	3	1	6	2	1
8	3	1	3	0	1
9	3	1	4	1	0
1	3	1	7	1	

From here on, the cycle repeats itself, since S_1 of the second cycle equals S_1 of the first cycle.
*Maximum required storage = 3 × 10⁶ m³.

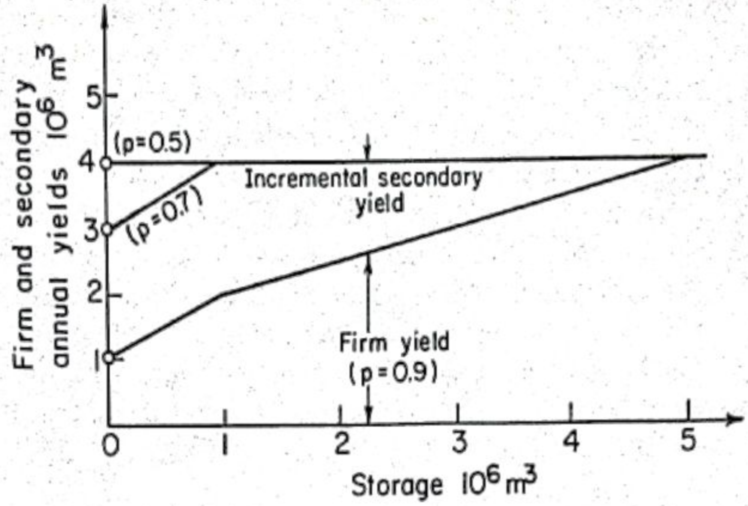


FIGURE 5-4
Storage-yield functions for various annual yields.

where Y_p is the incremental secondary yield if $p \neq n/(n+1)$, and

$$\alpha_{py} = \begin{cases} 1 & \text{if the rank } m \text{ of flow } \dot{I}_y \text{ is} \leq p(n+1) \\ 0 & \text{otherwise} \end{cases}$$

The other variables are as previously defined. The summation in Eq. (5-7) is over only those mean probabilities p of interest. As before, required storage is the maximum of all S_y values over twice the length of record.

Note that this method of calculating over-year storage requirements defines each secondary yield as an incremental yield. If, for example, $Y_{0.9}$ is the

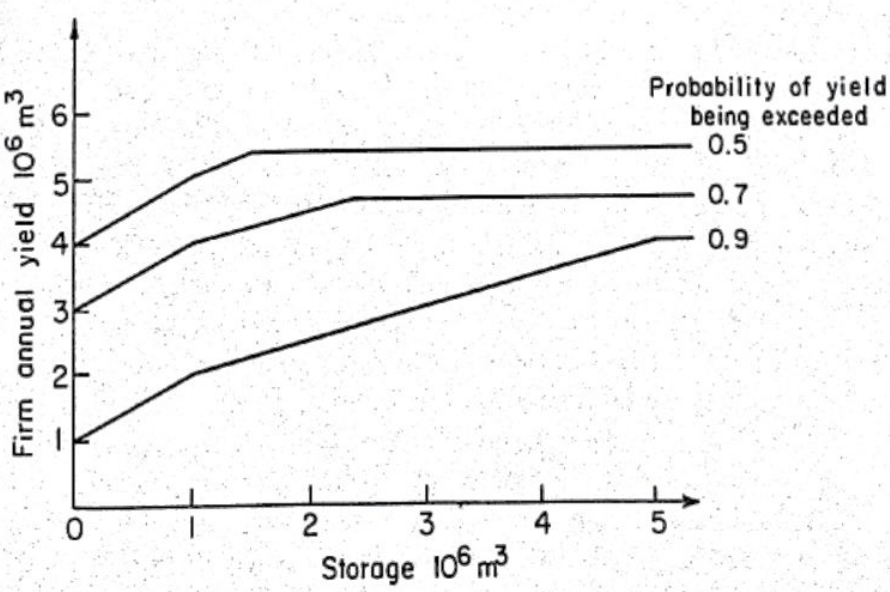


FIGURE 5-5
Some maximum firm and secondary annual yields for 9-year streamflow sequence.

firm yield, the incremental secondary yield of $Y_{0.8}$ is defined as the yield in excess of $Y_{0.9}$ that will be exceeded with a mean probability of 0.8. If, along with $Y_{0.9}$ and $Y_{0.8}$, an incremental secondary yield of $Y_{0.7}$ is defined, it will be the yield in addition to $Y_{0.9}$ plus $Y_{0.8}$ that has a mean probability of 0.7 of being exceeded. Each incremental secondary yield Y_p is the difference between the total secondary yield having a mean probability p and the total yield having the next highest mean probability that is defined. For example, if only two yields are defined, namely, a firm yield associated with a mean probability of 0.9 and a secondary yield associated with a mean probability of 0.7, and if the firm yield is 3 and the secondary yield is 4, then the variables $Y_{0.9}$ will equal 3 and $Y_{0.7}$ will equal 1. This is illustrated in Fig. 5-5. Assuming that these are the desired yields, the lower half of Table 5-1 illustrates the use of Eq. (5-7) for calculating the required storage.

Implicit in the derivation of these storage-yield functions is the assumption that the reservoir operating policy attempts to release a quantity of water at least equal to the annual inflow or firm yield, whichever is less, whenever possible. In other words, the magnitude of any particular annual firm yield is purposely decreased by always releasing the annual inflows that are less than the firm yield. Such a policy seems reasonable for two reasons. First, it is not likely that annual flows will be known with certainty, and this information would be needed if one were to store all the annual inflow during those years in which the firm yield could not be released, in order to maximize the annual firm yield or to minimize the probability of failing to meet the annual yield. Second, even if at the beginning of each year one could predict what the annual inflow would be, it is usually much more beneficial to have a larger number of smaller yield deficits than a smaller number of larger yield deficits.

The reservoir operating policy for secondary yield releases requires some forecast of the annual streamflow prior to the end of the year. The extent to which any forecast can be made, and the extent of the allowable yield deficit, should a deficit occur, can be reflected in the value of α_{py}, in Eq. (5-7), for those years y in which a yield failure may occur, i.e., in those years y having a flow I_y of rank $m > p(n + 1)$. The modified sequent-peak procedure defined by Eq. (5-7) assumes that no incremental secondary yield Y_p is released during those years in which a secondary yield failure may occur. This may be difficult or unreasonable to implement in actual practice. If so, those α_{py}'s that were set equal to 0 in Eq. (5-7) may be set equal to some fraction between 0 and 1, whatever seems reasonable for the particular situation. Again, these fractions will depend on both the ability to forecast future flows and the permissible or desired percentage deficit should a deficit occur. Note that the reliability of releasing a yield at least equal to $\alpha_{py}Y_p$, for any $\alpha_{yp} < 1$, is at most equal to the mean probability of the firm yield, namely, $n(n + 1)$.

In addition to the mass curves and storage-yield functions, linear programming techniques can also be used to calculate the over-year storage requirements associated with various yields Y_p. These techniques are based on the same operating policy assumptions as the algorithm just discussed. The required active reservoir capacity is defined as the minimum value that the maximum of all annual storage volumes can assume given a particular yield requirement. The annual storage volumes are derived for each year y from the continuity relationships between initial storage volumes S_y, the annual inflows I_y, the firm and incremental secondary yields Y_p, the excess releases, if any, R_y, and final storage volumes S_{y+1}:

$$S_y + I_y - \sum_{p \in P} \alpha_{py} Y_p - R_y = S_{y+1} \quad \begin{cases} y = 1,2,\ldots,n \\ \text{if } y = n,\ y+1 = 1 \end{cases} \tag{5-8}$$

where again, $\alpha_{py} = \begin{cases} 1 & \text{if flow } I_y \text{ has a rank } m \leq (n+1)p \\ <1 & \text{otherwise (0 will be assumed in the examples that follow)} \end{cases}$

and P is the set of exceedence probabilities p of interest in the analysis (e.g., 0.9 and 0.7, or 0.9, 0.8, and 0.5).

The objective of the model is to minimize $\max_y (S_y)$. This is done simply by minimizing a variable $S_0^{\max}$ whose value is no less than each storage volume S_y:

$$S_0^{\max} \geq S_y \qquad y = 1,2,\ldots,n \tag{5-9}$$

Again, assuming that the desired firm and incremental secondary yields $Y_{0.9}$ and $Y_{0.7}$ are 3 and 1 million m³, respectively, Table 5-2 lists an optimal solution to the linear programming model used to find the required storage. Note that the maximum required storage 3×10^6 m³ is identical to that shown by the storage-yield curves in Fig. 5-5.

It is possible that the use of a linear programming approach for defining over-year storage requirements for numerous reservoirs, each having a long record of available streamflows, could lead to a very large number of constraint equations (5-8) and (5-9). A little thought about when maximum storage vol-

Table 5-2 SAMPLE SOLUTION TO THE LINEAR PROGRAMMING STORAGE-YIELD MODEL

Year	Initial storage	Inflow	Firm yield	Incremental secondary yield	Excess release	Final storage
y	$\{S_y$	$+ I_y$	$- Y_{0.9}$	$- Y_{0.7}$	$- R_y\}$	$= S_{y+1}$
1	2	7	3	1	2	3*
2	3	3	3	1	0	2
3	2	5	3	1	0	3*
4	3	1	3	—	0	1
5	1	2	3	—	0	0
6	0	5	3	1	0	1
7	1	6	3	1	0	3
8	3	3	3	1	0	2
9	2	4	3	1	0	2

*Maximum storage required = 3×10^6 m³.

umes would be required for various sequences of streamflows may significantly reduce the number of constraint equations (5-9) that need be defined. The required number of equations (5-8) may also be reduced by first summing all continuous annual flow sequences that are greater than the maximum desired firm yield or mean annual flow, whichever is less; by next summing all remaining increasing or decreasing continuous sequences, excluding the minimum flows which have those mean probabilities p that are to be associated with the firm or secondary yields; and by finally summing all remaining flows that are for all practical purposes equal. When applied to problems involving more than one gage site, the grouping of various years of record must be the same at each site.

To illustrate how the data reduction procedure just described can assist in decreasing the size of the linear programming model of the 9-year example problem, the sequence of annual flows can be grouped in this order:

Year	1	2	3	4	5	6	7	8	9
Streamflow, 10^6 m³	7	3	5	1	2	5	6	3	4
Group	1	5	2	6	7	3		4	

Each flow in groups 1, 2, and 3 exceeds the mean annual flow 4×10^6 m³. Group 4 is the only remaining increasing flow sequence in which all flows are less than or equal to the mean annual flow and which does not include the lowest flows of record. There are no remaining decreasing flow sequences. Thus, this particular record does not permit much of a reduction in the number of continuity constraint equations (5-8). However, only three constraint equations (5-9) need be defined if there are no benefits associated with the excess releases or reservoir storage volumes. These three constraints would be for years 2, 4, and 8 when the initial reservoir storage is most likely to be required to meet any firm- or secondary-yield demands.

The resulting model for calculating the required over-year storage $S_0^{\max}$ is as follows:

Minimize $$S_0^{\max}$$

subject to
$$S_y + I_y - Y_{0.9} - Y_{0.7} - R_y = S_{y+1} \qquad y = 1,2,3$$
$$S_y + I_y - Y_{0.9} - R_y = S_{y+1} \qquad y = 4,5$$
$$S_y + (I_y + I_{y+1}) - 2(Y_{0.9}) - 2(Y_{0.7}) - R_y = S_{y+2} \qquad \begin{cases} y = 6,8; \\ \text{if } y = 8, \\ y + 2 = 1 \end{cases}$$
$$S_0^{\max} \geq S_y \qquad y = 2,4,8$$

Assuming that $Y_{0.9}$ and $Y_{0.7}$ are the only yields defined, having desired values of 3 and 1×10^6 m³, respectively, then one of the optimal solutions to the above model is the same as that shown in Table 5-2, except that the variables S_7, S_9, R_7, and R_9 are not defined. All optimal solutions require an over-year storage capacity $S_0^{\max}$ of 3×10^6 m³.

Thus far, evaporation and seepage losses have been ignored. To maintain linearity, it is customary to assume that estimates of these losses can be made by expressing the initial reservoir storage volume in every year as some fraction of the final volume in the previous year. Denoting ϵ as the fraction of the reservoir storage remaining after all losses from evaporation and seepage, constraint equations (5-8) become

$$\epsilon S_y + I_y - \sum_p \alpha_{py} Y_p - R_y = S_{y+1} \qquad \forall y \qquad (5\text{-}8a)$$

5-4.2 Within-Year Storage for Seasonal Yields

Additional storage is required if the distribution of streamflows within the year does not coincide with the desired within-year distribution of annual yields. To accurately predict both the over-year and within-year storage requirements using any of the methods just discussed, a simultaneous analysis of all successive streamflows would be required. For example, a linear programming model of a single reservoir having a monthly streamflow historical record of 20 years would require 20 × 12, or 240, continuity constraints to estimate both monthly and annual storage-yield relationships. These estimated relationships would be somewhat optimistic since such a model assumes the ability to forecast future streamflows.

Another estimate of the within-year storage requirements may be obtained by writing a single set of within-year continuity constraints, assuming average or mean within-year period inflows. These within-year constraints can then be included with the over-year constraints for computing the total active reservoir storage capacity. For a 12-period firm-yield allocation problem having 20 years of streamflow data, this approach requires only 12 plus 20 rather than 12 times 20 continuity constraints. This simplification may result in a within-year storage estimate that is slightly more or less than that actually needed, depending in part on the distribution of inflows and yield releases and, for multireservoir systems, on the efficiency of multiple reservoir operating policies. It is, therefore, imperative that the results be checked using simulation techniques.

The continuity constraints for each within-year period t are similar to the over-year constraints, Eq. (5-8), except that the mean within-year-period net inflows I_t and the within-year-period net firm and incremental secondary yields $[(1/n)\Sigma_y^n \alpha_{py}]y_{pt}$ replace the annual flows I_y and annual yields Y_p. Denoting s_t as the initial storage in period t and r_t as the excess release in period t, the within-year continuity constraints can be written

$$s_t + I_t - \sum_{p \epsilon P} \left(\frac{1}{n} \sum_y^n \alpha_{py}\right) y_{pt} - r_t = s_{t+1} \qquad t = 1,2,\ldots,T \qquad (5\text{-}10)$$

where $T + 1 = 1$ and for each probability $p \in P$,

$$\sum_{t}^{T} y_{pt} = Y_p \qquad (5\text{-}11)$$

To check for possible errors in the solution of any model, one can verify that the mean-annual excess release $(1/n)\Sigma_y^n R_y$ equals $\Sigma_t^T r_t$.

To facilitate the construction of reservoir operation rule curves (to be discussed later), an alternative and more conservative approach to within-year storage-capacity estimation is desired if more than one probability p is being considered. Since the within-year continuity constraints are solely for the purpose of identifying the storage required, if any, to redistribute the annual firm or incremental secondary yields Y_p in various periods t, some fraction of these annual yields can replace the inflow term in the above within-year continuity constraints. These fractions define the assumed inflow distribution of each Y_p. In the models that follow, the fraction of the annual yield Y_p flowing into the reservoir in each period t will be assumed to equal the mean unregulated flow in period t at the reservoir site divided by the mean annual flow. These mean flows will be based on only the flows in those years whose annual flows have a rank $m \leq (n + 1)p$. This fraction will be designated β_{pt}.

Defining for each period t within a year and for each mean probability p of interest, i.e., $p \in P$, the variables s_{pt} as the initial within-year storage, and y_{pt} as the firm- or incremental secondary-yield reservoir release, the within-year continuity constraints can be written

$$s_{pt} + \beta_{pt} Y_p - y_{pt} = s_{p,t+1} \qquad t = 1,2,3,\ldots,T \qquad (5\text{-}10a)$$

where $T + 1 = 1$. Such continuity constraints will ensure that the sum of the seasonal yields equals the annual yield for each mean probability p,

$$\sum_{t} y_{pt} = Y_p \qquad \forall p \in P \qquad (5\text{-}11a)$$

If desired, the within-year storage capacity $s_w^{\max}$ can be computed in a manner similar to the over-year storage capacity. The within-year capacity is the minimum value of $s_w^{\max}$, where $s_w^{\max} \geq \sum_{p \in P} s_{pt}$ for each period t. The maximum annual active storage capacity equals the over-year capacity $S_0^{\max}$ plus the within-year capacity $s_w^{\max}$. Where flood-control storage is to be considered, the definition of a fixed, annual active storage capacity may not be desirable, as will be discussed shortly.

5-4.3 Dead Storage

In addition to the active storage capacity required for over-year and within-year firm and secondary yields, additional storage volumes may be desired for increased recreational benefits, for increased hydropower heads, or for sediment accumulation. Hence, a dead storage variable S^{dead} can be introduced.

The value of this variable defines a minimum pool volume that can be added to the active and flood storage components in order to compute the total reservoir capacity.

5-4.4 Flood-Control Storage Capacity

The final component of reservoir storage is the capacity desired for the temporary storage of flood flows. Flood flows usually occur over periods lasting from a few hours up to a few days or weeks. Computational limitations prevent these short-duration flows from being explicitly defined in models that consider monthly or seasonal within-year periods. Nevertheless, various reservoir flood storage capacities that maximize the net expected economic benefits of reduced flood damages downstream can be estimated and evaluated in monthly or seasonal preliminary screening models. Of particular interest is the estimation of the optimal seasonal allocation of reservoir flood-control capacity and active storage for water supply within a fixed total reservoir capacity.

Flood-routing procedures, together with a knowledge of the flood-control operating policy and the channel storage characteristics between a reservoir site and a potential damage site downstream, make it possible to derive a functional relationship between the reservoir flood storage capacity and the reduction in the peak or crest of any particular flood at the potential damage site.

If discharges from the flood storage capacity of any reservoir can be controlled, the flood-control operating policy is initiated whenever the reservoir inflow exceeds the maximum nondamage release rate. The flood-control policy usually specifies the release $Q_{\text{out}\tau}$ at any time τ after the flood begins as some function of the inflow $Q_{\text{in}\tau}$ and the available flood storage capacity SF_τ:

$$Q_{\text{out}_\tau} = Q_{\text{in}_\tau} - f(SF_\tau) \cdot Q_{\text{in}_\tau} \qquad (5\text{-}12)$$

The function $f(SF_\tau)$ decreases nonlinearily from 1 to 0 as the available flood storage capacity decreases. Its optimal form can be found using simulation or other appropriate techniques that do not assume a perfect knowledge of the incoming flood hydrograph.

Unless a reservoir with sufficient flood storage capacity is immediately upstream of a potential damage site, reservoirs alone will not be able to completely eliminate damage should a flood occur. This is, in part, due to the portion of the flood peak contributed by the runoff between the upstream reservoir(s) and damage site. The portion of the flood peak that can be reduced at a given downstream site is a function of the flood-control operating policies of the upstream reservoirs, e.g., Eq. (5-12), and the flood storage capacity in each of these reservoirs.

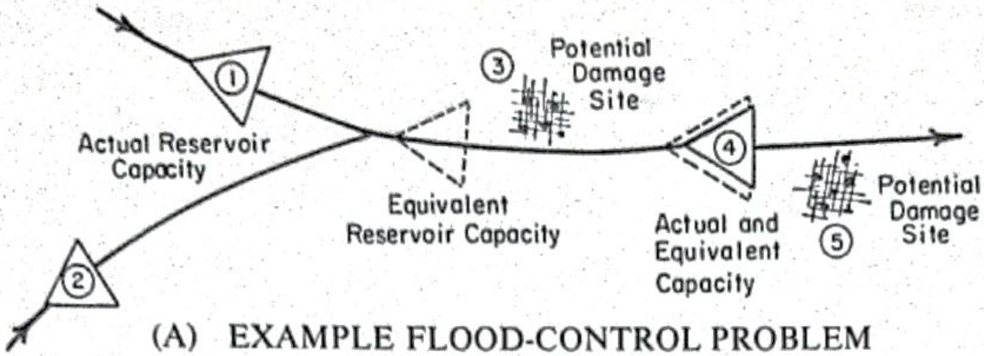
To overcome the analytical problem of defining and incorporating into a model multiple flood-peak or damage-reduction functions, each function depending on a particular mix of upstream reservoir flood storage capacities, the notion of an equivalent flood storage capacity immediately upstream of the potential damage site can be introduced. This equivalent capacity may be defined at any site in the basin whether or not such sites are potential reservoir sites.

The equivalent flood storage capacity at any given site is defined as the reservoir capacity that would reduce the flood peak at a downstream site the same amount as the flood and nonflood storage capacities of all upstream reservoirs. Since many of these upstream storage capacities are among the unknown variables in any long-run analysis of investment alternatives, relationships must be derived that define the equivalent storage at any particular site as functions of upstream actual and/or equivalent storage capacities. To derive each function requires the selection of one or more design floods (e.g., the standard project flood defined by the U.S. Army Corps of Engineers[5]), a flood-control operating policy [e.g., Eq. (5-12) or that resulting from fixed-capacity spillways or outlets] and flood-routing techniques.

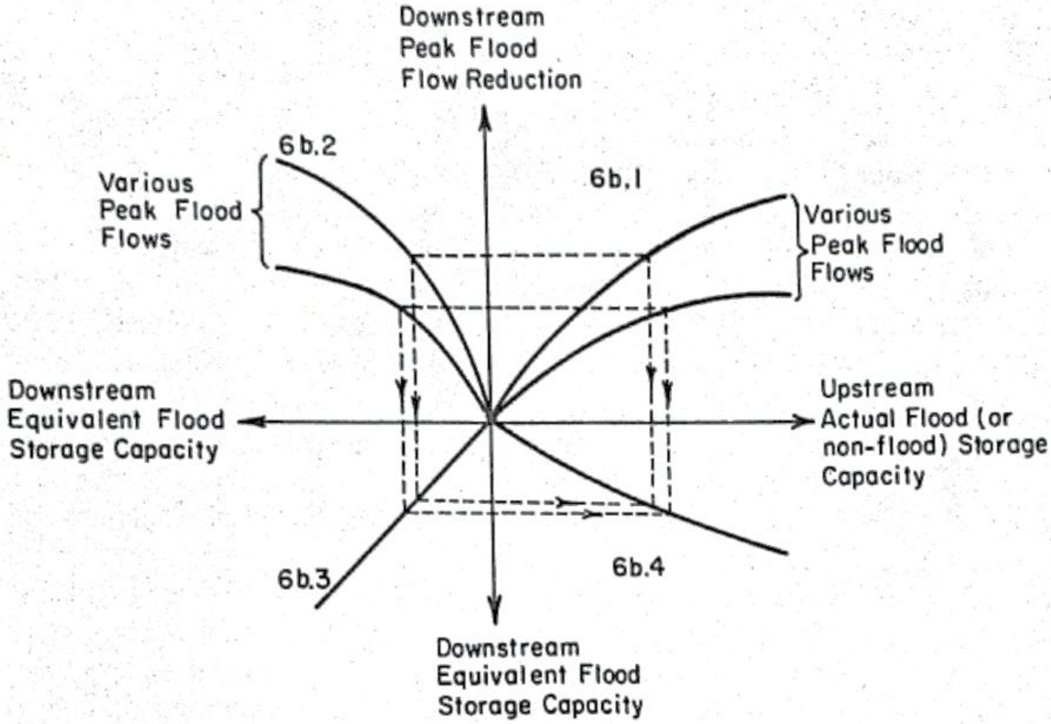
An example may help to illustrate how these equivalent flood-control capacity functions can be used to estimate desired flood-control capacities in a sequence of reservoirs. Referring to Fig. 5-6A, assume there are two potential damage centers, sites 3 and 5. Assuming that no reservoir capacity exists at sites 1, 2, and 3 for each period t (or groups of periods t if appropriate), two design floods are selected and routed through the basin to determine the unregulated flood hydrograph at sites 3 and 5. Next, for each design flood, various flood storage capacities are assumed at site 1 and the corresponding flood hydrographs at site 3 are calculated. Then, again assuming no flood storage capacity at site 1, the storage capacities and operating policy that would be required just upstream of site 3 to achieve the same flood-peak reduction at site 3 can be determined. A plot (Fig. 5-6B) of these corresponding flood storage capacities permits the estimation of the functional relationship between the flood storage capacity SF_t^1 at site 1 and the equivalent capacity EC_t^3 just upstream of site 3. Similarly, assuming no capacity at site 1, the equivalent flood storage capacities required just upstream of site 3 corresponding to various assumed capacities at site 2 can be calculated.

The individual functional relationships between each storage capacity SF_t^s $(s = 1,2)$ and the equivalent storage capacity EC_t^3 just upstream of site 3 can be denoted as $F_{s3t}(SF_t^s)$, or simply F (quadrant 6b.4). These functions F will equal 0 when SF_t^s is 0 and will be generally concave with slopes dF/dSF_t^s no greater than 1 and no less than 0. When these conditions are satisfied, the incremental storage capacity required just upstream of site 3 to reduce a flood peak some specified amount will not exceed the incremental capacity required further upstream (e.g., at site 1 or at site 2).

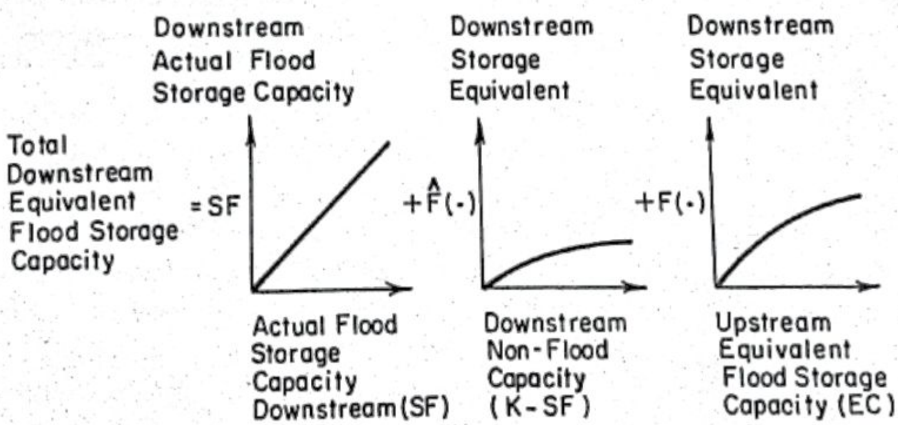
Concave equivalent capacity functions having slopes dF/dSF_t^s no greater than 1 imply that the peak flow reduction at a potential damage site resulting from any particular upstream reservoir flood storage capacity increases as the distance between the reservoir site and the potential damage site decreases. However, this may not be true if reservoir discharge capacities are uncontrolled or are fixed by the capacity of the outlet or spillway. A downstream res-



(A) EXAMPLE FLOOD-CONTROL PROBLEM



(B) DERIVATION OF DOWNSTREAM EQUIVALENT STORAGE CAPACITY



(C) COMPONENTS OF TOTAL DOWNSTREAM EQUIVALENT FLOOD STORAGE CAPACITY

FIGURE 5-6
An illustration of how equivalent flood-control capacity functions can be used to estimate desired flood-control capacities in a sequence of reservoirs.

ervoir (e.g., just upstream of site 3 in Fig. 5-6A) may require more capacity to achieve a given flood-peak reduction at a downstream potential damage site (site 3) than would a reservoir farther upstream. This results from accumulation of a substantial portion of the inflow hydrograph that precedes the peak flow, and the volume in this leading portion becomes larger as the flood travels downstream. In these situations, $dF/dSF_t^s > 1$ for some range of SF_t^s.

Concave approximations of any convex portion of these equivalent capacity functions will simplify model solution but may lead to conservative solutions, i.e., more flood storage capacity than is desired. The preliminary screening procedure will then involve reducing these capacities, a far easier task than bringing additional capacity into the solution.

If only actual flood storage capacities SF_t^s are used to derive equivalent capacities, another conservative error may be introduced by not considering the reduction in peak flows resulting from water stored in multipurpose reservoirs. Lakes formed by water volumes stored in reservoirs widen the flood channels and hence attenuate the peak flood flows.

The flood storage capacity (at sites $s = 3$ or 4 in Fig. 5-6A) just upstream of a potential damage site (sites $s' = 3$ or 5, respectively) that reduces the peak flood flow the same amount as a volume of water S_t^s stored upstream (e.g., at sites $s = 1, 2,$ or 4) can be obtained using the same procedures required to estimate equivalent capacity as a function of upstream actual capacity. These procedures are illustrated in Fig. 5-6. Once obtained, the equivalent flood storage-capacity functions $F_{ss't}(SF_t^s)$ and $\hat{F}_{ss't}(S_t^s)$ can be approximated by a series of linear segments for incorporation into linear programming models. Several techniques for doing this will be reviewed later in the chapter.

Before discussing each of the constraints needed to model the problem illustrated in Fig. 5-6A, it is necessary to select some substitute variable to replace the actual dead plus active storage volume S_t^s. This is necessary because the value of S_t^s is random and hence cannot be predicted at the time of a flood (which is also random). Obviously, the selection of a nonrandom substitute variable is a matter of judgment. One possibility might be to replace S_t^s with the total nonflood storage capacity, i.e., the total capacity K^s less the flood storage capacity SF_t^s, $(K^s - SF_t^s)$. An actual storage volume of this amount would result in the widest lake and hence the greatest flood-peak attenuation due to channel storage. However, anything less than this volume would mean that some of the active or dead storage capacity can be used for flood storage capacity (an event that usually occurs). Hence, using the total nonflood storage capacity, an unknown but nonrandom variable, seems as good a procedure as any, and probably a conservative one in most cases.

Returning to the example illustrated in Fig. 5-6A, an initial estimate of the equivalent flood-control capacity just upstream of site 3 equals the sum of the two pairs of functions $F_{s3t}(SF_t^s)$ and $\hat{F}_{s3t}(K^s - SF_t^s)$ of flood storage capacity and nonflood storage capacity,

$$EC_t^3 = \sum_{s=1}^{2} [F_{s3t}(SF_t^s) + \hat{F}_{s3t}(K^s - SF_t^s)] \qquad \forall t \qquad (5\text{-}13)$$

even though no reservoir is to be constructed at that site 3.

The extent that EC_t^3 defined by Eq. (5-13) may be greater or less than the actual equivalent capacity corresponding to a particular combination of flood and nonflood storage capacity will depend on the flood-control operating policy and also on the magnitude of these capacities if both pairs are greater than zero. Hence, once some initial values for SF_t^s and $K^s - SF_t^s$ are obtained, the actual equivalent flood storage capacity EC_t^3 should be computed to determine the appropriate constant α_{3t} to multiply the right-hand side of Eq. (5-13) by prior to obtaining another, hopefully more accurate, solution:

$$EC_t^3 = \alpha_{3t} \sum_{s=1}^{2} [F_{s3t}(SF_t^s) + \hat{F}_{s3t}(K^s - SF_t^s)] \qquad \forall t \qquad (5\text{-}13a)$$

This iterative procedure can continue until sufficient accuracy is obtained for the purposes of preliminary screening prior to a more detailed simulation analysis.

The equivalent flood-control capacity at site 3 having been estimated, similar procedures can be used to define the equivalent flood-control capacity at site 4. Since site 4 is a potential reservoir site, the equivalent flood-control capacity at that site is equal to the actual flood storage capacity SF_t^4 at that site plus the sum of that portion contributed by the equivalent capacity just upstream of site 3, $F_{34t}(EC_t^3)$, and the nonflood storage capacity, $\hat{F}_{44t}(K^4 - SF_t^4)$ (see Fig. 5-6C):

$$EC_t^4 = F_{34t}(EC_t^3) + SF_t^4 + \hat{F}_{44t}(K^4 - SF_t^4) \qquad \forall t \qquad (5\text{-}14)$$

Procedures for estimating flood storage capacities in this example problem can be used for any configuration of reservoir sites and potential flood-damage centers.

The equivalent flood-control capacity EC_t^s just upstream of any particular potential damage site s in period t having been estimated, whether or not an actual or potential reservoir exists, there remains the need for deriving the functional relationships $BF_t^s(EC_t^s)$ between the expected annual flood-damage reduction and the equivalent flood-control capacity at each potential damage site s in each period t. Once the expected flood-reduction benefits are defined as functions of equivalent flood-control capacities EC_t^s, and once the costs are defined as functions of the actual reservoir storage capacities, these economic benefit and cost functions can be included in the objective function of the model to find the set of actual flood capacities SF_t^s in each period t that maximize the expected net flood-control benefits.

Figure 5-7 shows one method of calculating the expected annual flood-reduction benefits as a function of upstream equivalent flood storage capacity.

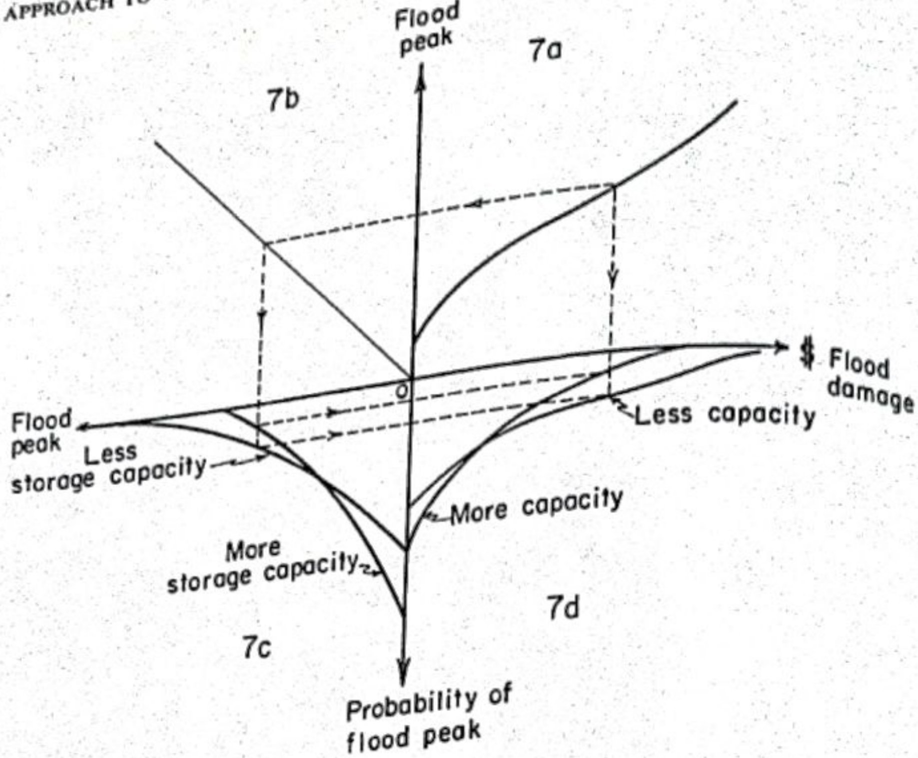


FIGURE 5-7
Derivation of flood-damage probability functions.

The information shown in quadrants 7a and 7c is derived from data that include records of historical and/or simulated floods, channel capacities, existing or potential flood-plain development, and the flood-control reservoir operating policy. (The question of optimum flood-plain development and its relation to the probability of flooding is not answered here.) For each equivalent flood storage capacity there will exist a unique flood-peak probability distribution, as illustrated in quadrant 7c, and, therefore, a unique damage-probability function, as shown in quadrant 7d. Each computed damage-probability function in quadrant 7d, when integrated, provides one point on the expected flood-damage reduction curve of Fig. 5-8. For example, if the probability functions in quadrants 7c and 7d are exceedence probabilities, then the area under each damage-probability function in quadrant 7d is the annual expected flood damage associated with the particular reservoir flood storage capacity. The difference between this value and the expected annual damage without additional reservoir capacity is the expected flood-damage reduction and, hence, a point on the function illustrated in Fig. 5-8. Deriving these damage probability functions for a number of equivalent flood storage capacities makes it possible to estimate the functional relationship between the expected flood-damage reduction downstream and the equivalent flood-control storage immediately upstream.

The above procedure admittedly requires a considerable amount of data gathering and analysis. Yet it provides at least an approximate means of incor-

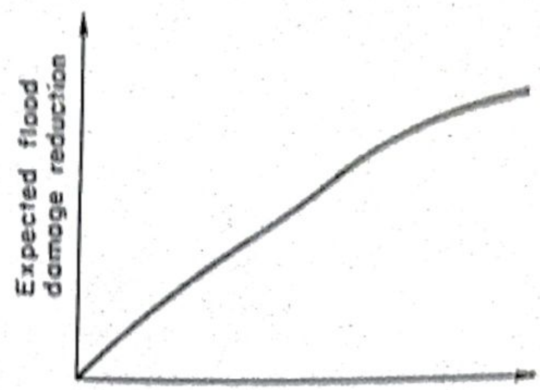


FIGURE 5-8
Expected flood-damage reduction function.

porating investment alternatives for flood control into a management model having as its smallest unit of time a month or even a season. The length of each within-year period t must be such that the expected flood damages that may occur in the period are independent of any damages that may have occurred in previous periods. If the periods t defined for the within-year allocation of water supplies are such that the expected flood damages in each period are not independent of those in previous periods, then various periods t may be appropriately combined for the purposes of flood-control analysis.

5-4.5 Total Reservoir Capacity

Now that each component of reservoir storage has been identified, all that remains is to add them together to define the total reservoir capacity K at any particular site. Recall that the over-year capacity $S_o^{\max}$ and the dead storage S^{dead} were not functions of each period t. Within-year storage s_{pt} and flood-control capacity SF_t were defined for each period t. This was done to permit the additional storage of water supplies during periods when floods are unlikely. Conversely, during periods of high flood risk and low water-supply requirements, it could be beneficial to use more of the total reservoir capacity for flood control and less for water supply. Significant costs savings may result if trade-offs between within-year storage and flood-control capacity can be made throughout the year. The trade-offs are not possible if the total reservoir capacity is assumed to be the sum of the maximum within-year storage $s_w^{\max}$, maximum flood storage capacity $SF^{\max}$, maximum over-year storage $S_o^{\max}$, and dead storage S^{dead} variables.

Permitting trade-offs between the active storage and flood-control capacities, the total capacity K of each reservoir is defined by the set of constraints

$$K \geq S^{\text{dead}} + S_o^{\max} + \sum_{p \in P} s_{pt} + SF_t \qquad \forall t \tag{5-15}$$

A portion of the overall objective is to minimize the total annual capital and OMR costs $C(K)$ associated with the reservoir capacity K.

5-5 FLOOD PROTECTION AND DAMAGE REDUCTION

In the previous section on reservoir storage, methods were outlined for estimating the flood storage capacities, if any, at each reservoir site in each within-year period, that maximized the expected net annual flood-damage reduction benefits. In this section these methods will be extended and modified somewhat (1) to include the possibility of levees or dikes and other channel-capacity improvements at each potential damage site; (2) to provide a cost-effective approach for estimating structural flood-control alternatives that will ensure a prespecified, but not necessarily equal, level of flood protection at each potential damage site; and (3) to select among the various discrete levels of protection the most economically efficient protection level at each potential damage site. If neither channel-capacity improvements nor specific levels of flood protection are of concern, and only reservoir storage capacity is to be considered for flood-peak reduction, then the methods previously discussed are sufficient, and the modeling approach proposed in this section is not required.

In this section, as in the last, only structural flood-control measures will be analyzed. The modeling techniques proposed for the preliminary screening of structural flood-control measures will not be as detailed as would be appropriate, and feasible, if the problem were simply one of single-purpose flood control. Instead, the methodology is relatively simple and can be easily incorporated into a more general multipurpose surface-water investment and management model. Simplicity requires numerous assumptions, and the effect of these assumptions on the accuracy of the solution should be evaluated before any final recommendation is made.

In particular situations, some changes in the suggested modeling approach may be required or desirable. Any changes or assumptions that are made in modeling specific river basin systems should be done in a manner that will achieve the purpose of the analysis (e.g., for preliminary screening) as simply and as effectively as possible.

5-5.1 A Cost-Effective Analysis

Any level of flood protection can be expressed as the expected return period of the maximum flood peak for which no damage will occur. For convenience, let p denote the reciprocal of the expected return period y. This is the probability that, in any single year, a flood equal to or greater than that expected once every y years will occur. [Note that the probability of a once-in-y-year flood being equaled or exceeded in an n-year period is $1-(1-1/y)^n$. The exceedence probability is dependent on n and is $p = 1/y$ only when $n = 1$.]

The unregulated natural peak flow (cubic meters per second) at a potential damage site can be either reduced by upstream reservoir flood storage capacity and/or contained within the channel at the potential damage site by levees and other channel-capacity improvements.

Referring to the example illustrated in Fig. 5-6A, suppose that potential damage sites 3 and 5 require p_3 and p_5 levels of flood protection. If the reservoir at site 4 were not to be built, then the structural measures available to achieve the required protection levels at sites 3 and 5 include the flood storage capacities at the upstream reservoir sites 1 and 2 and channel-capacity improvements at sites 3 and 5. Hence, to protect to the levels required, the corresponding peak flows at the potential damage sites must be either reduced by upstream reservoir storage (converted to equivalent capacity at site 3) or contained within the channel at those potential damage sites 3 and 5.

Denote $QN^s_{tp_s}$ as the unregulated or natural peak flow in period t at site s having a one-period probability of exceedence of p_s. To protect site s from this peak flow, a portion $QS^s_{tp_s}$ of the peak flow may be reduced by upstream flood storage capacity. The remaining peak flow $QR^s_{tp_s}$ must be contained within the channel at site s:

$$QN^s_{tp_s} = QS^s_{tp_s} + QR^s_{tp_s} \qquad \forall t,\ s = 3,5 \tag{5-16}$$

For a peak flood flow defined by a particular one-period probability of exceedence p, the storage capacity at any upstream site required for any flood-peak reduction at a downstream potential damage site can be obtained by simulation. Such a relationship is shown in Fig. 5-6B, quadrant 6b.1. The required equivalent storage capacity EC^3_t at site 3 will be a convex function $R_{33p_3}(QS^3_{tp_3})$ or $R_{35p_5}(QS^5_{tp_5})$ of the peak flow reduction $QS^s_{tp_s}$. The functions $R(\cdot)$ are dependent on the location and the distance between the equivalent reservoir at site 3 and the potential damage sites 3 and 5, and on the magnitude p_s of the peak flood flows being considered at sites 3 and 5. These convex functions, or piecewise linear approximations of them, also define the maximum peak reductions associated with various maximum peak flows that can be obtained from upstream reservoir flood-control capacities.

Continuing to assume that the reservoir at site 4 in Fig. 5-6A is not to be built, then the equivalent capacity EC^3_t at site 3 must be the larger of the equivalent capacity requirements for sites 3 and 5:

$$EC^3_t \geq R_{33p_3}(QS^3_{tp_3}) \qquad \forall t \tag{5-17}$$

$$EC^3_t \geq R_{35p_5}(QS^5_{tp_5}) \qquad \forall t \tag{5-18}$$

If the equivalent capacity at reservoir site 4 were included in the model, Eq. (5-18) would be replaced by Eq. (5-18*a*):

$$EC^4_t \geq R_{45p_5}(QS^5_{tp_5}) \qquad \forall t \tag{5-18a}$$

The equivalent capacities at sites 3 and 4 are, of course, functions of the actual flood and nonflood storage capacities at sites 1, 2, and 4 [Eqs. (5-13) and (5-14)]. The actual flood storage capacities will be a portion of the total reservoir capacities K^s at those sites [Eq. (5-15)], which in turn will determine the annual reservoir costs $C^s(K^s)$ that are included in the objective function of the model.

Also to be included in the objective function are the annual costs of channel-capacity improvements, if any, at the potential damage sites $s = 3$ and 5. The channel-capacity costs are those required to contain the maximum $QR^s_{p_s}$ of the remaining peak flows $QR^s_{tp_s}$ in each period t:

$$QR^s_{p_s} \geq QR^s_{tp_s} \qquad \forall t,\ s = 3,5 \tag{5-19}$$

The annual cost of channel-capacity improvements will be a function $CF^s\,(QR^s_{p_s})$ of the maximum peak flow $QR^s_{p_s}$ at each potential damage site s. This generally convex function can be made piecewise linear for inclusion in the objective function. [Note that Eqs. (5-16) and (5-19) may be combined by replacing the $QR^s_{tp_s}$ term in Eq. (5-16) with $QR^s_{p_s}$ and changing the equality to a $\leq$ inequality. If this is done, Eq. (5-19) is not required.]

The modeling procedure just described is one that minimizes the annual costs to ensure a prespecified level of flood protection at each potential damage site. It assumes that upstream channel improvements do not alter the peak flood flows at downstream potential damage sites. Depending on the distance between damage sites, this assumption may or may not be appropriate. Hence, this, as well as other assumptions required to obtain equivalent storage capacities, should be checked using more precise simulation techniques once a set of structural investment alternatives has been identified by these screening models.

5-5.2 A Benefit-Cost Analysis

It is possible to extend the cost-effective approach to one that considers more than a single flood-protection level at each potential damage site. This will permit an analysis of the expected flood reduction benefits, as well as of the flood-control costs associated with each level of protection.

The expected flood damage-reduction benefits associated with any level of flood protection is dependent on the particular mix of flood storage and channel capacities used to achieve this level of protection. Figure 5-7 illustrates the influence of changes in flood storage capacity on the flood peak-probability function (quadrant 7c) and hence on the expected flood-damage reduction. Channel capacity alters the flood peak-damage function (quadrant 7a). Both change the flood damage-probability function (quadrant 7d) and consequently the expected flood damage-reduction benefits.

Rather than attempting to define explicitly in a single function the interdependent relationships between flood-damage benefits and flood storage and channel capacity, an estimate of the expected annual flood-control benefits BF^s_p associated with any level of flood protection p at site s may be made. This estimate can be based on a reasonable combination of structural flood-control measures. Once the model is solved and the best level of protection is identified, then the actual flood damage-reduction benefits (resulting from the particular mix of structural flood-control measures identified in the model solution) can be compared to the estimated expected flood damage-reduction benefits BF_p^s. If the difference is substantial, then the estimated BF_p^s can be changed and the model resolved. Methods for deriving the annual expected damage-reduction benefits from quadrant 7d were discussed in the previous section.

To identify the particular level of flood protection p_s at each potential damage site s that maximizes the total expected net flood reduction benefits, an integer decision variable $X_{p_s}^s$ can be introduced. If at a particular site s, p_s^* is the desired level of protection, then $X_{p_s^*}^{s*}$ will equal 1; all other $X_{p_s}^s$ $(p_s \neq p_s^*)$ will equal 0. Since only one, if any, level of flood protection p_s can be provided at each potential damage site s,

$$\sum_{p_s} X_{p_s}^s \leq 1 \qquad \forall s \tag{5-20}$$

and

$$X_{p_s}^s = 0,1 \qquad \forall s, p_s \tag{5-21}$$

This integer variable $X_{p_s}^s$ will be multiplied by the estimated annual expected flood damage-reduction benefits $BF_{p_s}^s$, and each term $X_{p_s}^s BF_{p_s}^s$ will be included in the objective function of the model. Equations (5-20) and (5-21) will ensure that only one $BF_{p_s}^s$ for each site s will contribute to the value of the objective function.

The variable X_p^s having been defined, Eq. (5-16) can now be written for each discrete level of protection p_s of interest at each site s:

$$X_{p_s}^s QN_{tp_s}^s = QR_{tp_s}^s + QS_{tp_s}^s \qquad \forall s, p_s, t \tag{5-22}$$

Again, Eqs. (5-20) and (5-21) ensure that no more than one Eq. (5-22) will apply at each site s. Hence, only one channel-capacity cost function $CF^s(QR_{p_s}^s)$ will be nonzero for each site s, even though channel-capacity cost functions for each flood-protection level p_s of interest are included in the objective function of the model. Equations (5-17), (5-18) or (5-18a), and (5-19) remain the same.

5-6 SURFACE-WATER ALLOCATIONS

Major categories of surface-water allocations include domestic, commercial, and industrial water supplies; irrigation and other agricultural supplies; hydroelectric energy production; flow augmentation for navigation and quality control; and reservoir-based recreation. As previously discussed, the expected net benefits derived from allocations to each of these uses are in part dependent on the expected or target allocations and on the dependability of the allocations. In this section, the mathematical expressions defining the allocations of firm or secondary yields, and/or storage volumes when appropriate, to each of these use categories will be developed for inclusion in an overall surface-water model.

5-6.1 Nonagricultural Water Supplies

Domestic, commercial, and industrial water users often base their investments on the availability of firm yields. Secondary yields may also be of some benefit for activities that are not as essential or for which no significant losses will occur if there are more frequent shortages in supply.

If the expected net benefits associated with a predetermined allocation of an annual firm or secondary yield to a particular domestic, commercial, or industrial use can be reasonably quantified, then a benefit-loss function similar to that shown in Fig. 5-1 can be used to estimate the desired annual target-yield allocation and any losses associated with shortages from the desired or prespecified within-year allocations of that annual target yield. Assume that the desired allocation of any annual target yield T_p to each period t within the year is defined by a fraction δ_{pt} of the annual target. Hence, for each yield having a mean probability p of being exceeded,

$$\sum_t \delta_{pt} = 1 \qquad \forall p \tag{5-23}$$

and $\delta_{pt} T_p$ is the within-year period allocation of an annual target yield T_p.

The allocation of firm and incremental secondary yields to meet the demands of each water user cannot exceed the quantity y_{pt} of yields available at each use site. Furthermore, the reliability of those allocated yields must be no less than the desired reliability of the target demands, T_p. Thus at each use site, for each mean probability p' of interest in each period t,

$$\sum_{p \geq p'} y_{pt} \geq \sum_{p \geq p'} \delta_{pt} T_p - D_{pt} + E_{pt} \tag{5-24}$$

where D_{pt} is the deficit allocation and E_{pt} is the surplus or excess allocation, if any. Equation (5-24) permits the allocation of yields having relatively higher reliabilities to meet user demands requiring relatively lower reliabilities, but not vice versa. In cases where it is clear that each yield component y_{pt} would not be used to satisfy demands requiring less reliability, then Eq. (5-24) can be simplified by eliminating the summations:

$$y_{pt} \geq \delta_{pt} T_p - D_{pt} + E_{pt} \qquad \forall p \in P, t \tag{5-24a}$$

The objective, in part, is to maximize the benefits $B_p(T_p)$ derived from the annual target allocation of T_p less the short-run losses associated with deficit, $LD_{pt}(D_{pt})$, or surplus, $LE_{pt}(E_{pt})$, if any:

$$\text{Maximize} \qquad \sum_p \{B_p(T_p) - \sum_t [LD_{pt}(D_{pt}) + LE_{pt}(E_{pt})]\} \tag{5-25}$$

For water users having no economic benefit or loss data, constraints specifying minimum acceptable annual or within-year period allocations can be used as a substitute means of estimating the system costs, i.e., the change in net benefits to other water users, associated with the various constrained allocations.

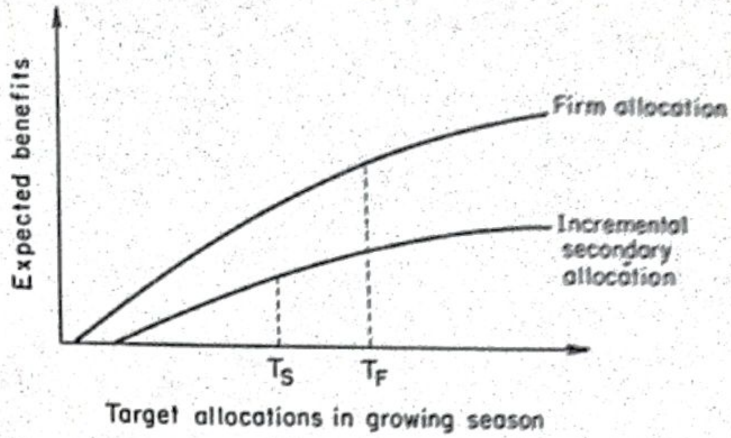


FIGURE 5-9
Irrigation benefit functions for firm and incremental secondary yields.

5-6.2 Agricultural Water Use

A detailed analysis of irrigation investment alternatives must consider numerous input factors that are required for the production and harvest of irrigated crops. Included among the important factors are the amount and distribution of capital and equipment, labor, land, and water throughout the growing season. If any one of these factors is sufficiently lacking at any time during the growing season, the crops may die. If this occurs, it is obvious that additional allocations of these input factors in subsequent periods will not be able to restore them. Hence, one of the important considerations in the development of irrigation models is the necessity to incorporate the dependency of the net benefits derived during the harvest period to the distribution of resources allocated in each of the previous periods of the growing season.

River basin models that include the possibility of allocating surface water to various irrigation sites and to numerous other users cannot be as detailed as models that examine only irrigation investment and operating policies at a particular irrigation site. Nevertheless, detailed models for each specific irrigation site that include all important input factors, specific soil types and cropping patterns, variable rainfall, variations in yield associated with variations in the allocated amount and distribution of water, and the like[6,7,8,9] can be used to advantage in developing the functional relationships between the amount and distribution of water allocated to a particular irrigation site and the maximum expected net benefits derived during the harvest period. Clearly, these benefits are also dependent on the reliability of the allocations of water during the growing season. Such functions might be similar to those shown in Fig. 5-9.

If detailed irrigation analyses are not possible, the net benefit functions illustrated in Fig. 5-9 can be estimated given some information on the area of land that can be irrigated per unit of irrigation water allocated, the quantity of each irrigated crop that can be produced per unit area of land that is irrigated, the total fixed and variable costs of producing each crop, and the unit price that will clear the market of any quantity of each crop to be produced. For example, consider a single crop and a single soil type. Let T_p be the unknown irrigation target allocation of water having a mean probability p of being exceeded. Define H as the hectares of land that can be irrigated for each unit of T_p and c as the quantity of the crop that can be grown and harvested per hectare of irrigated land. The total quantity of the crop produced is therefore cHT_p, which will be denoted as q_p. This quantity q is unknown since T_p is unknown. If the unit price is specified by the function $P(q)$ and the total fixed and variable costs of production are C_F and $C_V(q)$, respectively, then a conservative estimate of the expected net benefits (sum of consumers and producers surplus) can be defined as follows:

$$NB(q_p) = p\left[\int_0^{q_p} P(q)\,dq - C_V(q_p)\right] - C_F \tag{5-26}$$

This estimate of the net benefits $NB(q_p)$ is conservative since it assumes that when T_p cannot be allocated in full (the probability of which is $1 - p$) no allocation will be made, and hence, the net benefits will be negative and equal to the fixed costs. Recall from the discussion beginning on page 170 that, in the case of firm- or secondary-yield allocations, the water management model developed in this chapter assumes that whenever deficits must occur, they can be limited to a certain predetermined percentage of the yields, at least $100(n)/(n + 1)$ percent of the time (where n is the number of years of streamflow records). Hence, while the model does not assume the ability to forecast when deficits will occur in the future, it can provide some estimate of the maximum extent of most of the deficits when they do occur. Consequently, the analyst could make some assumption about the extent of the net benefits obtained during most of those seasons when the target demand cannot be fully met. However, for preliminary screening, the conservative assumptions incorporated into Eq. (5-26) seem as reasonable as anv.

To extend this example a step further, assume that the unit price is defined by the linear function $p_o - bq$, where $b \geq 0$. Then from Eq. (5-26) the expected net benefits will equal

$$NB(q_p) = p\,[p_o q_p - 0.5bq_p^2 - C_V(q_p)] - C_F \tag{5-27}$$

Since the corresponding marginal revenue function is $p_0 - 2bq$, the producers, expected net revenue is obtained from Eq. (5-27) by replacing the coefficient 0.5 by 1.0. Both expressions are concave functions similar to those shown in Fig. 5-9. This concavity is not dependent on a linear unit-price function. Nonlinear decreasing unit-price functions $P(q)$ would also result in concave net benefit or net revenue functions, all of which can be incorporated into water management models.

The net benefit or producers net revenue functions just described are deterministic in the sense that the unit price is assumed to be known given the quantity of crop produced. This may not be the case, especially from an individual producer's point of view. If some estimates can be made of the probability of the demand for various quantities given the unit-price function $P(q)$, then these probabilities can be incorporated into the construction of expected net benefit or net revenue functions.

Once the functions illustrated in Fig. 5-9 are estimated, they provide a basis for incorporating irrigation into the water management model. Referring to Fig. 5-9, if the total firm and incremental secondary target allocations at a particular irrigation site equal $T_F + T_S$ units of water, the expected net benefits will equal $B_F(T_F) + B_S(T_S)$, provided that the distribution of this allocation during the growing season is within specified limits required for each crop being produced. These limits can be expressed as fractions of the total allocation.

Define T_{Ft} and T_{St} as those portions of the total firm irrigation target T_F and incremental secondary-yield target T_S allocated in period t of the growing season, and $A_t^{\min}$ and $A_t^{\max}$ as the minimum and maximum fractions of the total allocation allowed in period t. The irrigation objective is to

$$\text{Maximize} \qquad B_F(T_F) + B_S(T_S) \tag{5-28}$$

subject to the following constraints for each period t within the growing season:

1 Water distribution constraints:

$$(A_t^{\min})(T_F + T_S) \leq T_{Ft} + T_{St} \leq (A_t^{\max})(T_F + T_S) \tag{5-29}$$

2 Constraints ensuring that the amount allocated does not exceed the available streamflow yield:

$$\begin{aligned} y_{Ft} &\geq T_{Ft} \\ y_{St} &\geq T_{St} \end{aligned} \tag{5-30}$$

3 Constraints equating the sum of each periods' allocation to the total allocation in the growing season:

$$\begin{aligned} \sum_t T_{Ft} &= T_F \\ \sum_t T_{St} &= T_S \end{aligned} \tag{5-31}$$

5-6.3 Hydropower Production

The production of hydroelectric energy at any reservoir site in each period t is dependent on the number of hours in the period h_t; the installed plant capacity p; the flow through the turbines Q_t; the average storage head H_t, the load factor L_t, and plant factor P_t; and a constant k for converting the product of flow, head, and plant efficiency to kilowatt-hours of energy.

The energy is kilowatt-hours KWH_t produced in period t can be derived from a knowledge of the plant efficiency e and the productive storage head and flow. Assume that the head is measured in meters and the flow in millions of cubic meters. Since 1 m^3 of water weighs 10^3 kg, Q_t million m^3 of water falling through a productive head of H_t releases $10^9\ Q_t H_t$ kg·m of energy. Each kilogram-meter of energy is equivalent to 2.723×10^{-6} KWH. It follows that the kilowatt-hours of energy produced in period t equals

$$\text{KWH}_t = 2723 Q_t H_t e \qquad \forall t \tag{5-32}$$

If the flow is measured in units of acre-feet and the head in feet, then the same derivation leads to the production function

$$\text{KWH}_t = 1.024 Q_t H_t e \qquad \forall t \tag{5-33}$$

Hydroelectric energy is divided into firm energy and surplus, or secondary, energy. Firm energy is that amount that can be supplied virtually at all times. It is a contractual term defining the energy having an assured availability to the consumer to meet all or any agreed upon portion of his load requirements. Surplus energy is all available energy in excess of the firm power. It is energy that is generated by water that could not be stored or conserved to meet future firm energy commitments. Surplus power produced with no guarantee as to continuity of service is often called dump energy.

Assuming a conservative value for the firm productive head in each period t, estimates of the firm energy production can be made by substituting for the flow variable Q_t the firm reservoir release y_{pt}, where $p = n/(n+1)$:

$$\text{KWH}_t^{\text{firm}} \le kH_t y_{pt} \qquad \forall t \tag{5-34}$$

Similarly the surplus energy can be estimated from the incremental secondary yields:

$$\text{KWH}_t^{\text{surplus}} \le kH_t \left(\sum_p y_{pt}\right) \qquad \forall t,\ p < n/(n+1) \tag{5-35}$$

The amount of energy produced also depends on the installed capacity of the plant, as well as on the load and plant capacity factors. In the analysis of investment alternatives for water-resource systems, the plant capacity factor P_t and the load factor L_t, being characteristics of the power system supply and demand, respectively, are assumed known.

The plant factor is a measure of power plant use. It is defined as the ratio of the average load on the plant for the period of time considered to the installed plant capacity. It may or may not vary in each period t. The total energy produced cannot exceed the product of the plant factor, P_t, the number of hours in the period h_t, and the plant capacity P, measured in kilowatts:

$$\text{KWH}_t^{\text{firm}} + \text{KWH}_t^{\text{surplus}} \le P_t h_t P \qquad \forall t \tag{5-36}$$

The load factor L_t is an index of the distribution of the load demanded over the period t which may vary for each period. It is defined as the ratio of the average firm load to the peak firm load in the period. The production of firm energy by all hydroelectric plants in the system is limited to no more than that fraction of the time in each period specified by the load factor. Stated differently, the total firm energy produced in each period t cannot exceed the product of the load factor, the number of hours in the period, and the total plant capacity. Hence, for all sites s having hydroelectric plants of capacity P^s,

$$\sum_s \text{KWH}_t^{\text{firm}\,s} \leq L_t H_t \sum_s P^s \qquad \forall t \tag{5-37}$$

For example, a load factor of 0.20 implies that if a firm flow of Y_p is to be used to produce energy, it can be used during only 20 percent of the period. The lower the load factor the greater will be the required total storage and/or total plant capacity. Equation (5-37) is a constraint on the entire hydroelectric system, and unless that system is completely defined within the model, the constraint may not be applicable.

Subject to the above constraints, with the possible exception of (5-37), the objective of this portion of the overall surface-water model is to maximize the expected benefits derived from firm and surplus power production less the annual net costs of installed capacity $C(P)$:

$$\text{Maximize} \qquad \sum_t \left[B_t^F(\text{KWH}_t^{\text{firm}}) + B_t^S(\text{KWH}_t^{\text{surplus}})\right] - C(P) \tag{5-38}$$

While the above expected benefit functions should be based on the mean probabilities associated with the terms firm and surplus, they may not adequately account for the extent of the benefits or losses when the firm and secondary yields fail, since the magnitude of any shortages in yields or storage heads are not known before the model is solved. Therefore, it is imperative that any assumptions made regarding storage heads and net expected benefits be compared to the values of the decision variables after model solution. Several trial solutions may be required prior to obtaining a solution that has sufficient consistency and accuracy for the purposes of the preliminary screening of alternative solutions.

5-6.4 Flow Augmentation and Navigation

The benefits of flow augmentation, whether for quality improvement, for shoreline management, or for navigation, are usually not well defined. Nevertheless, it may be desirable to estimate the reduction in those quantifiable benefits throughout the system if streamflows at various sites s within the river basin are constrained to be no less than some minimum amount $Q_{pt}^{\min s}$ at least p percent of the time in any period t. This can be done by constraining the appropriate streamflow yields y_{pt}^s to be no less than $Q_{pt}^{\min s}$:

$$y_{pt}^s \geq Q_{pt}^{\min s} \qquad \text{for appropriate } s, p, t \tag{5-39}$$

As discussed in Chap. 6, the effects of flow augmentation on stream quality may or may not be beneficial. An analysis of the surface-water quality associated with various constrained firm and secondary streamflow yields should provide information that, together with system costs, can be used to select the appropriate minimum yields $Q_{pt}^{\min}$.

5-6.5 Reservoir Recreation

The benefits derived from recreation activities on a reservoir depend in part on the target storage volumes for which recreation facilities are built and the short-run losses resulting from deviations from the planned or target volumes. An approximate estimate of the average storage volume in any particular period t is the sum of the dead storage volume S^{dead}, the average over-year storage volume $(1/n)\ \Sigma_y^n S_y$ [or if grouped annual flows are used, $(1/2n)\ \Sigma_g \eta_g (S_g + S_{g+1})$, where η_g is the number of years in each group g], and the average expected within-year period volume $\Sigma_{p \epsilon P}\ p(s_{pt} + s_{p,t+1})/2$. Letting T be the target storage volume for recreation and D_t and E_t the deficit or excess storage volume, if any, then,

$$T - D_t + E_t = \frac{1}{2n} \sum_g \eta_g (S_g + S_{g+1}) + S^{\text{dead}} + \frac{\sum_{p \epsilon P} p(s_{pt} + s_{p,t+1})}{2} \tag{5-40}$$

The objective of this portion of the model is to maximize the annual expected net benefits $B(T)$ associated with a recreation volume target T less the losses $LD_t(D_t)$ and $LE_t(E_t)$ due to deficit or excess volumes, if any.

Maximize
$$B(T) - \sum_t [LD_t(D_t) + LE_t(E_t)] \tag{5-41}$$

5-7 MODEL SYNTHESIS, SOLUTION, AND EVALUATION

The remainder of this chapter will be devoted to a brief review of how each of the model components discussed above can be combined into an investment model of a multireservoir, multipurpose surface-water system. Once this is done the model can be solved and its solution further analyzed and simulated in order to evaluate and improve its accuracy.

The partially hypothetical river basin planning problem illustrated in Fig. 5-10 will be used to demonstrate model development, solution, and evaluation. Although simplified, the example problem will serve to illustrate how models can be developed for any river basin configuration and how they can be used to analyze multipurpose investment and operating policy alternatives.

The unregulated streamflows at any point in the river basin, illustrated in Fig. 5-10, are assumed to be some multiple of the unregulated streamflows at one of the two gage sites, G^1 or G^9. These multiples are given in Table 5-3, along with the fraction of the allocation to consumptive uses that is assumed to be returned to the stream.

Economic efficiency, without regard to net benefit redistribution or non-monetary goals, would require maximizing the total expected benefits from all water uses less the sum of the losses due to allocation shortages or surpluses and the costs of investments. Table 5-4 lists the components of the efficiency objective for each water-use site in the example problem. Recall that each vari-

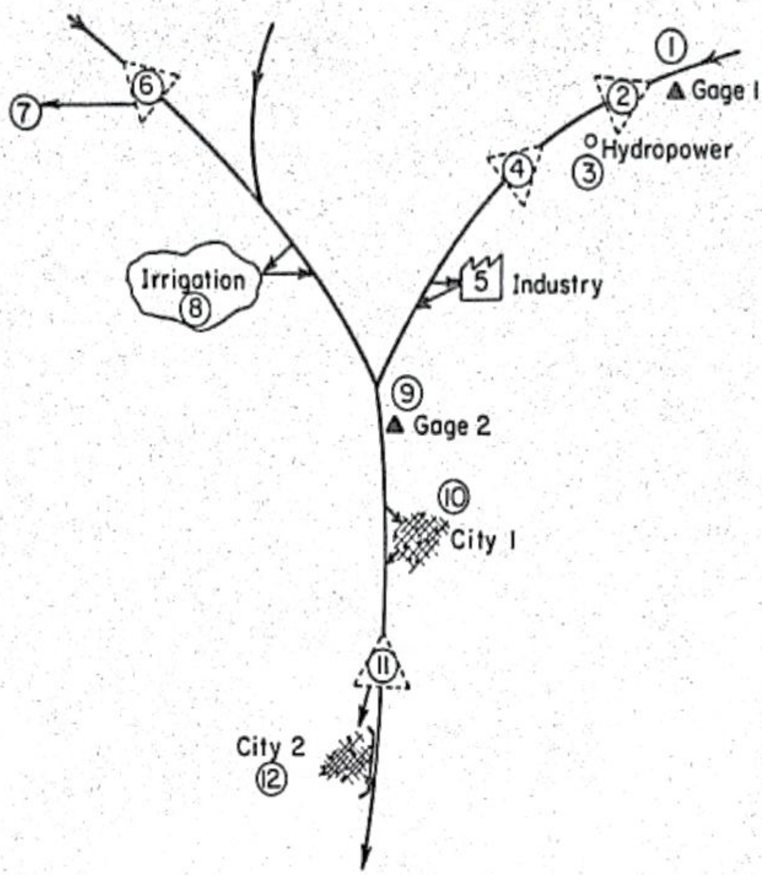


FIGURE 5-10
Example river basin planning problem.

Table 5-3 HYDROLOGIC DATA

Site	Purpose	Unregulated flow as multiple of gage flow	Fraction of allocation returned to stream
1	Gage 1	$1.0G^1$	—
2	Reservoir	$1.2G^1$	—
3	Hydropower	$1.2G^1$	1.0
4	Reservoir	$1.6G^1$	—
5	Industry	$1.8G^1$	0.7
6	Reservoir	$0.4G^9$	—
7	Diversion	$0.4G^9$	0.0
8	Irrigation	$0.6G^9$	0.4
9	Gage 2	$1.0G^9$	—
10	City 1	$1.2G^9$	0.8
11	Reservoir	$1.3G^9$	—
12	City 2	$1.3G^9$	0.8

Table 5-4 OBJECTIVE FUNCTION COMPONENTS FOR EXAMPLE PROBLEM (FIG. 5-10)

Overall objective:

Maximize $\sum_j W_j \left(\sum_s A_{js} NB_s\right) = \begin{cases} \text{total weighted } W_j \text{ annual net benefits } NB_s \text{ at each site } s \text{ allocated} \\ A_{js} \text{ to each user group } j\text{: } \left(\sum W_j = 1;\ \sum_j A_{js} = 1 \quad \forall s\right) \end{cases}$

Site s	Annual net benefit components for each site s	
2	$NB_2 = -C^2(K^2)$	= Reservoir costs
3	$NB_3 = \sum_t [B_t^{F3}(\text{KWH}_t^{F3}) + B_t^{S3}(\text{KWH}_t^{S3})] - C^3(P^3)$	= Firm and secondary hydroelectric energy benefits less capacity costs
4	$NB_4 = -C^4(K^4)$	= Reservoir costs
5	$NB_5 = \sum_p \{B_p^5(T_p^5) - \sum_t [LD_{pt}^5(D_{pt}^5) + LE_{pt}^5(E_{pt}^5)]\}$	= Industry target water-supply benefits less losses from deficits or gains from excesses
6	$NB_6 = -C^6(K^6)$	= Reservoir costs
7	$NB_7 = \sum_p \{B_p^7(T_p^7) - \sum_t [LD_{pt}^7(D_{pt}^7) + LE_{pt}^7(E_{pt}^7)]\}$	= Diversion target water-supply benefits less losses from deficits or gains from excesses
8	$NB_8 = \sum_p B_p^8(T_p^8)$	= Irrigation benefits from firm and incremental secondary target water-supply allocations
10	$NB_{10} = \sum_p \{B_p^{10}(T_p^{10}) - \sum_t [LD_{pt}^{10}(D_{pt}^{10}) + LE_{pt}^{10}(E_{pt}^{10})]\} + \sum_t BF_t^{10}(EC_t^{10})$	= City 1 target water-supply benefits less losses from deficits plus gains from excesses plus expected annual flood reduction benefits
11	$NB_{11} = B^{11}(T^{11}) - \sum_t [LD_t^{11}(D_t^{11}) + LE_t^{11}(E_t^{11})] - C^{11}(K^{11})$	= Recreation storage target benefits less losses from deficits or excesses and reservoir costs
12	$NB_{12} = \sum_p \{B_p^{12}(T_p^{12}) - \sum_t [LD_{pt}^{12}(D_{pt}^{12}) + LE_{pt}^{12}(E_{pt}^{12})]\} + \sum_p X_p^{12} BF_p^{12} - \sum_p CF_p^{12}(QR_p^{12})$	= City 2 target water-supply benefits less losses from deficits or gains from excesses, plus flood protection benefits less channel-capacity costs

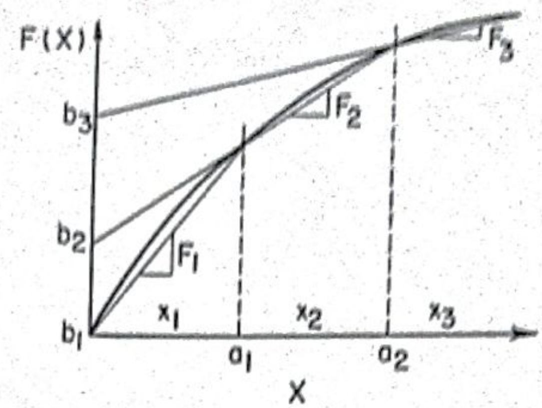
able K is the total reservoir capacity, T is a target, P is the hydroelectric plant capacity, D is a deficit allocation, E is a surplus allocation, and EC is the equivalent flood storage capacity. The functions $B(\cdot)$, $BF(\cdot)$, $LD(\cdot)$ and $LE(\cdot)$, and $C(\cdot)$ are the long-run benefit, short-term loss (or gain, if negative), and cost functions, respectively. The subscript t refers to a within-year period, and the superscripts or subscripts F, S, and p refer to firm or incremental secondary yields having a mean probability p of being exceeded.

In this example problem, flood protection will be required at the two city sites, 10 and 12. At site 10, only upstream reservoir flood storage capacity will be considered for flood-damage reduction. At site 12, a number of discrete levels of protection (against floods having a 1-year probability of exceedence of p) will be considered. Associated with each discrete protection level p are the expected annual flood damage-reduction benefits BF_p^{12} and a 0,1 variable X_p^{12} that identifies the optimal level of protection. The variable X_p^{12} also equates to zero the benefits and costs of other levels of protection. Costs of flood protection include expenditues required for reservoir flood storage capacity and for channel-capacity improvement.

Note that, at site 12, the subscript p related to flood control is not the same as the subscript p related to water-supply allocations. The former signifies a 1-year probability of exceedence of peak flood flows and the latter a mean probability of exceedence for water-supply yields.

The objective function components listed in Table 5-4, if made piecewise linear, would require segment definition and bounds. Figure 5-11 reviews three procedures for doing this. Linear programming, using approximation methods (a) or (b), can be used if the function $F(X)$ in Fig. 5-11 is to be maximized. Note that method (b) requires no additional constraints for increased accuracy, only an increase in the number of variables w_t. Yet method (b) cannot be used if the concave function $F(X)$ is to be minimized. For minimization of concave functions (or maximization of convex functions), separable programming applied to method (a) will ensure that each allocation x_l equals its upper bound $a_l - a_{l-1}$ before $x_{l+1} > 0$, but there is no guarantee that a global minimum will be obtained. To guarantee a global minimum, the mixed integer programming method (c) can be used as a solution technique. [Note that any function of the form $\Sigma_l F_l x_l$ using method (a) is equivalent to $\Sigma_l \Sigma_j^l F_j(a_j - a_{j-1})w_l$ using method (b) and $\Sigma_l(b_l + F_l \hat{x}_l)z_l$ using method (c). Details of these linearization and solution procedures can be found in most texts in operations research.] Efficient computer programs for solving linear programming, separable programming, and mixed integer programming problems are readily available at most scientific computer facilities throughout much of the world.

In addition to the constraints defining piecewise linear segments, bounds, and maximum or minimum allocations, capacities, or targets, the remainder of the constraints simply define the continuity of surface water throughout the basin and the interrelationships between the decision variables. As has been the case throughout this chapter, the mean probabilities of exceedence p



Method (*a*):

$$\text{Optimize } F(X) \simeq F_1x_1 + F_2x_2 + F_3x_3 + \cdots$$
$$\text{Subject to } X = x_1 + x_2 + x_3 + \cdots$$
$$x_1 \leq a_1$$
$$x_2 \leq a_2 - a_1$$
$$x_3 \leq a_3 - a_2$$
$$\vdots$$

Method (*b*):

$$\text{Maximize } F(X) \simeq F(a_1)w_1 + F(a_2)w_2 + F(a_3)w_3 + \cdots$$
$$\text{Subject to } X = a_1w_1 + a_2w_2 + a_3w_3 + \cdots$$
$$w_1 + w_2 + w_3 + \cdots \leq 1$$

Method (*c*):

$$\text{Minimize } F(X) \simeq b_1z_1 + F_1\hat{x}_1 + b_2z_2 + F_2\hat{x}_2 + b_3z_3 + F_3\hat{x}_3 + \cdots$$
$$\text{Subject to } X = \hat{x}_1 + \hat{x}_2 + \hat{x}_3 + \cdots$$
$$\hat{x}_1 \leq a_1z_1$$
$$\hat{x}_2 \leq a_2z_2$$
$$\hat{x}_3 \leq a_3z_3$$
$$\vdots$$
$$z_1 + z_2 + z_3 + \cdots \leq 1$$
all z_i integer

FIGURE 5-11
Piecewise linearization procedures.

included in the constraints are only those mean probabilities of interest, i.e., those included in the set *P*.

Before proceeding with the identification of all the constraints required to construct a model of the basin shown in Fig. 5-10, it is advantageous to modify slightly the over-year and within-year reservoir storage-continuity equations (5-8) and (5-10). These modifications will not result in additional constraints, but they will facilitate the interpretation of the results, especially for multireservoir basins.

Recall that over-year storage is required to *increase* annual yields Y_p; hence, the flows into each reservoir need only be those annual unregulated flows *in excess* of the existing yields that are contained in the total annual unregulated inflows at the reservoir site. It is desirable, therefore, in each year y, to subtract from each total annual inflow to a reservoir that portion of the inflow having the same mean probabilities of exceedence as the release yields Y_p included in the over-year continuity constraint for that year y (namely, those annual yields associated with coefficients $\alpha_{py} = 1$).

The sum of the firm and incremental secondary yields contained in the unregulated annual flow I_y^s at a reservoir or control site s in year y is simply the particular annual unregulated flow at site s having a rank m equal to one plus the number of years of record, $n + 1$, times the lowest mean probability of exceedence p included in the over-year continuity constraint for that year y. This total-yield component of the unregulated annual flow at each site s in each year y will be designated U_y^s. Note that this yield component U_y^s of each annual unregulated flow I_y^s will never exceed the annual flow; i.e., $I_y^s \geq U_y^s$ for all years y and sites s. In those years having an annual unregulated flow of rank m equal to $n + 1$ times the lowest mean probability of exceedence included in the over-year continuity constraint (i.e., the minimum p for which $\alpha_{py} = 1$), the sum of the firm and incremental yields U_y^s, if any, will equal the unregulated annual flow I_y^s. Otherwise, the unregulated annual flow will exceed U_y^s.

For example, consider two reservoir sites, an upstream site $s - 1$ and a downstream site s. At these sites the annual unregulated flows for each year y are I_y^{s-1} and I_y^s. The unregulated interflow between these two sites is $I_y^s - I_y^{s-1}$. For each year y the difference $U_y^s - U_y^{s-1}$ will define the sum of the firm and incremental yields contained within the unregulated interflow. The sum of the downstream reservoir release yields, $\Sigma_{p \epsilon P} \alpha_{yp} Y_p^s$, will be those in addition to the existing interflow yields, if these interflow yields $U_y^s - U_y^{s-1}$ are subtracted from the unregulated interflow $I_y^s - I_y^{s-1}$ entering the downstream reservoir.

While it is sufficient to define annual uncontrolled yields for the purposes of identifying the over-year storage requirements, if any, at each reservoir site, these uncontrolled yields along with the regulated yields Y_p eventually must be allocated to each of the within-year periods t specified in the model. In addition, each firm- or incremental secondary-yield component must be identified at each control site throughout the basin. This can be done by first defining a variable V_p^s to equal the total-yield component of the unregulated annual flow at site s having a mean probability p of exceedence. This total-yield component is simply the annual unregulated flow at site s whose rank m equals $(n + 1)p$. (Note that if p is the lowest mean probability of exceedence included in the over-year continuity equation for year y, then $V_p^s = U_y^s$.)

The total yield V_p^s at each site s is the sum of all firm- and incremental secondary-yield components having mean probabilities of exceedence equal or greater than p. Each of these components can be designated v_p^s, and are equal to V_p^s minus V_{p+}^s, where $p+$ is the next highest mean probability of interest. If p is the maximum mean probability of interest, i.e., max $p \epsilon P$, then v_p^s is the firm-yield component of the unregulated annual flow. Otherwise, it is an incremental secondary-yield component.

Still needed before it is possible to compute the within-year storage requirements is an estimate of how the expected or average annual flows are distributed among each period. As previously defined, the proportion β^s_{pt} of the annual unregulated yield v^s_p and of the additional annual regulated yield Y^s_p (resulting from over-year storage, if any) that is naturally available in each period t at each site s is assumed to be the same proportion as the mean unregulated flow in period t is to the mean unregulated annual flow in the same period t at that site s. These mean unregulated flows are based on only those years in which the rank m of the annual unregulated flows is less than or equal to $(n+1)p$; hence, the need for the subscript p in the fraction β^s_{pt}. The expected within-year yields at any reservoir site s in period t will equal $\beta^s_{pt}(v^s_p + Y^s_p)$, unless within-year storage is available for redistribution.

The following set of constraints for the problem defined in Fig. 5-10 will help to illustrate how these modifications can be made, as well as showing how the various model components discussed in this chapter can be combined into a model of an entire basin. The constraints will be developed for each site (except gage sites 1 and 9) identified in Fig. 5-10.

Site 2: Reservoir

1. *Over-year storage capacity* (assuming no grouping of flows which, if done using the procedures previously discussed, may considerably reduce the number of constraints). For each year y:

$$\epsilon^2 S^2_y + 1.2(G^1_y - U^1_y) - \sum_{p \epsilon P} \alpha_{yp} Y^2_p - R^2_y = S^2_{y+1}$$

$$\begin{bmatrix}\text{initial}\\ \text{storage}\\ \text{less}\\ \text{losses}\end{bmatrix} + \begin{bmatrix}\text{inflow less}\\ \text{unregulated}\\ \text{yields}\end{bmatrix} - \begin{bmatrix}\text{additional}\\ \text{yield}\\ \text{releases}\end{bmatrix} - \begin{bmatrix}\text{excess}\\ \text{release}\end{bmatrix} = \begin{bmatrix}\text{final}\\ \text{storage}\end{bmatrix}$$

$$S^{\max}_0 \geq S^2_y$$

$$\begin{bmatrix}\text{over-year}\\ \text{capacity}\end{bmatrix} \geq \begin{bmatrix}\text{over-year}\\ \text{storage}\end{bmatrix}$$

[The 0,1 variable α_{yp} is as defined for Eq. (5-7) on page 168. It equals 1 whenever the mean probability p is greater than or equal to the rank m of G^1_y divided by $n+1$.]

2. *Within-year storage*. For each period t and mean probability p:

$$s^2_{pt} + \beta^2_{pt}(Y^2_p + v^2_p) - y^2_{pt} = s^2_{p,t+1}$$

$$\begin{bmatrix}\text{initial}\\ \text{storage}\end{bmatrix} + \begin{bmatrix}\text{expected}\\ \text{regulated and}\\ \text{unregulated}\\ \text{yield inflow}\end{bmatrix} - \begin{bmatrix}\text{firm- or}\\ \text{incremental}\\ \text{secondary-}\\ \text{yield}\\ \text{release}\end{bmatrix} = \begin{bmatrix}\text{final}\\ \text{storage}\end{bmatrix}$$

3. *Reservoir capacity*. For each period t:

$$K^2 \geq S^{\text{dead}^2} + S_0^{\max^2} + \sum_{p \in P} s_{pt}^2 + SF_t^2$$

[total capacity] ≥ [dead storage] + [over-year storage capacity] + [within-year storage] + [flood storage capacity]

Site 3: Hydropower

1. *Firm energy production*. For each period t and $p = n/(n+1)$:

$$\text{KWH}_t^{F^3} \leq kH_t^2 y_{pt}^2$$

[firm energy] ≤ [function of head and firm yield]

2. *Surplus energy production*. For each period t:

$$\text{KWH}_t^{S^3} \leq kH_t^2 \sum_p y_{pt}^2$$

[surplus energy] ≤ [function of head and incremental secondary yields, i.e., those having $p < n/(n+1)$]

3. *Plant capacity limitations*. For each period t:

$$\text{KWH}_t^{F^3} + \text{KWH}_t^{S^3} \leq P_t^3 h_t P^3$$

[total energy] ≤ [plant factor] × [hours in period] × [plant capacity].

Site 4: Reservoir

1. *Over-year storage capacity*. For each year y:

$$\epsilon^4 s_y^4 + 0.4(G_y^1 - U_y^1) + R_y^2 - \sum_{p \in P} \alpha_{py} Y_p^4 - R_y^4 = S_{y+1}^4$$

[initial storage less losses] + [interflow between sites 2 and 4 less unregulated yields] + [upstream reservoir excess releases] − [additional yield releases] − [excess release] = [final storage]

$$S_0^{\max^4} \geq S_y^4$$

[over-year capacity] ≥ [over-year storage]

2. *Within-year storage*. For each period t and mean probability p:

$$s_{pt}^4 + \beta_{pt}^4 (Y_p^4 + v_p^4 - v_v^2) + y_{pt}^2 - y_{pt}^4 = s_{p,t+1}^4$$

[initial storage] + [expected inflow of unregulated and regulated yields in period t: $v_p^4 - v_p^2 = 0.4\, v_p^1$] − [firm- or incremental secondary-yield release] = [final storage]

3. *Equivalent flood capacity*. For each period t:

$$EC_t^4 = \alpha_{4t}\,[\mathrm{F}_{24t}(SF_t^2) + SF_t^4 + \hat{F}_{24t}(K^2 - SF_t^2) + \hat{F}_{44t}(K^4 - SF_t^4)]$$

$$\begin{bmatrix}\text{equivalent}\\ \text{capacity}\end{bmatrix} = \begin{bmatrix}\text{functions of flood and non-}\\ \text{flood storage capacities}\end{bmatrix}$$

4. *Reservoir capacity*. For each period t:

$$K^4 \geq S^{\text{dead}^4} + S_0^{\max^4} + \sum_{p\epsilon P} s_{pt}^4 + SF_t^4$$

$$\begin{bmatrix}\text{total}\\ \text{capacity}\end{bmatrix} \geq \begin{bmatrix}\text{dead}\\ \text{storage}\end{bmatrix} + \begin{bmatrix}\text{over-year}\\ \text{storage}\\ \text{capacity}\end{bmatrix} + \begin{bmatrix}\text{within-}\\ \text{year}\\ \text{storage}\end{bmatrix} + \begin{bmatrix}\text{flood}\\ \text{storage}\\ \text{capacity}\end{bmatrix}$$

Site 5: Industry

1. *Allocation limitations*. For each period t and mean probability p:

$$\beta_{pt}^5\,(v_p^5 - v_p^4) + y_{pt}^4 \geq \delta_{nt}^5 T_v^5 - D_{pt}^5 + E_{pt}^5$$

$$\begin{bmatrix}\text{yield in}\\ \text{uncontrolled}\\ \text{interflow:}\\ v_p^5 - v_p^4 = 0.2v_p^5\end{bmatrix} + \begin{bmatrix}\text{yield}\\ \text{released}\\ \text{from up-}\\ \text{stream}\\ \text{reservoir}\end{bmatrix} \geq \begin{bmatrix}\text{within-year}\\ \text{period target}\end{bmatrix} - [\text{deficit}] + [\text{surplus}]$$

Site 6: Reservoir

1. *Over-year storage capacity*. For each year y:

$$\epsilon^6 S_y^6 + 0.4\,(G_y^9 - U_y^9) - \sum_{p\epsilon P} \alpha_{yp} Y_p^6 - R_y^6 = S_{y+1}^6$$

$$\begin{bmatrix}\text{initial}\\ \text{net}\\ \text{storage}\end{bmatrix} + \begin{bmatrix}\text{inflow less}\\ \text{unregulated}\\ \text{yields}\end{bmatrix} - \begin{bmatrix}\text{annual ad-}\\ \text{ditional}\\ \text{firm- and}\\ \text{incremental}\\ \text{secondary-}\\ \text{yield re-}\\ \text{leases}\end{bmatrix} - \begin{bmatrix}\text{excess}\\ \text{release}\end{bmatrix} = \begin{bmatrix}\text{final}\\ \text{storage}\end{bmatrix}$$

$$S_0^{\max^6} \geq S_y^6$$

$$\begin{bmatrix}\text{over-year}\\ \text{capacity}\end{bmatrix} \geq \begin{bmatrix}\text{over-year}\\ \text{storage}\end{bmatrix}$$

2. *Within-year storage*. For each period t and mean probability p:

$$s_{pt}^6 + \beta_{pt}^6\,(Y_p^6 + v_p^6) - y_{pt}^6 = s_{p,t+1}^6$$

$$\begin{bmatrix}\text{initial}\\ \text{storage}\end{bmatrix} + \begin{bmatrix}\text{expected regulated}\\ \text{and unregulated}\\ \text{yield inflow}\end{bmatrix} - \begin{bmatrix}\text{firm- or}\\ \text{incre-}\\ \text{mental}\\ \text{secondary-}\\ \text{yield re-}\\ \text{lease}\end{bmatrix} = \begin{bmatrix}\text{final}\\ \text{storage}\end{bmatrix}$$

3. *Reservoir capacity*. For each period t:

$$K^6 \geq S^{\text{dead}^6} + S_0^{\max^6} + \sum_{p\epsilon P} s_{pt}^6 + SF_t^6$$

$$\begin{bmatrix}\text{total}\\ \text{capacity}\end{bmatrix} \geq \begin{bmatrix}\text{dead}\\ \text{storage}\end{bmatrix} + \begin{bmatrix}\text{over-year}\\ \text{capacity}\end{bmatrix} + \begin{bmatrix}\text{within-}\\ \text{year}\\ \text{storage}\end{bmatrix} + \begin{bmatrix}\text{flood}\\ \text{storage}\\ \text{capacity}\end{bmatrix}$$

Site 7: Diversion

Diversion allocation. For each period t and mean probability p:

$$y_{pt}^{6} \geq \delta_{pt}^{7} T_{p}^{7} - D_{pt}^{7} + E_{pt}^{7}$$

[available yield] ≥ [target diversion] − [deficit] + [surplus]

Site 8: Irrigation

1. *Firm- and incremental secondary-yield allocations.* For each period t in the growing season and mean probability p:

$$\beta_{pt}^{8}(v_{p}^{8} - v_{p}^{6}) + y_{pt}^{6} - (\delta_{pt}^{7} T_{p}^{7} - D_{pt}^{7} + E_{pt}^{7}) \geq T_{pt}^{8}$$

[yield in uncontrolled interflow: $v_p^8 - v_p^4 = 0.2v_p^8$] + [upstream reservoir yield] − [upstream withdrawal] ≥ [firm- or incremental secondary-yield irrigation target]

$$A_{t}^{\min}\left(\sum_{p \in P} T_{p}^{8}\right) \leq \sum_{p \in P} T_{pt}^{8} \leq A_{t}^{\max}\left(\sum_{p \in P} T_{p}^{8}\right)$$

[minimum proportion of annual target yield to be allocated in period t] ≤ [within-year period t target allocations] ≤ [maximum proportion of annual target yields to be allocated in period t]

2. *Total allocations.* For each probability p of interest

$$\sum_{t} T_{pt}^{8} = T_{p}^{8}$$

[total within-year allocations] = [total growing-season allocation]

Site 10: City 1

1. *Allocation of water supply.* For each period t and mean probability p:

$$\beta_{pt}^{10}(v_{p}^{10} - v_{p}^{4} - v_{p}^{6}) + y_{pt}^{4} + y_{pt}^{6}$$

[firm or secondary interflow yield between sites 4, 6, and 10: $v_p^{10} - v_p^6 - v_p^4 = 0.8v_p^9 - 1.6v_p^1$] + [firm or secondary yields from upstream reservoirs]

$$- (\delta_{pt}^{7} T_{p}^{7} - D_{pt}^{7} + E_{pt}^{7}) - 0.6\,(T_{pt}^{8}) - 0.3\,(\delta_{pt}^{5} T_{p}^{5} - D_{pt}^{5} + E_{pt}^{5})$$

− [diversion allocation] − [irrigation consumption] − [industry consumption]

$$\geq \delta_{pt}^{10} T_{p}^{10} - D_{pt}^{10} + E_{pt}^{10}$$

≥ [target allocation] − [deficit] + [surplus]

2. *Equivalent flood storage capacity.* For each period t:

$$EC_{t}^{10} = \alpha_{10,t}\{F_{6,10,t}(SF_{t}^{6}) + F_{4,10,t}(EC_{t}^{4}) + \hat{F}_{6,10,t}(K^{6} - SF_{t}^{6})\}$$

[equivalent capacity] = [functions of actual and equivalent flood and nonflood storage capacities upstream]

Site 11: Reservoir

1. *Over-year storage capacity.* For each year y:

$$\epsilon^{11} S_y^{11} + 0.9\,(G_y^9 - U_y^9) - 1.6\,(G_y^1 - U_y^1) + R_y^4 + R_y^6$$

[initial storage less losses] + [unregulated interflow less firm and incremental yields] + [excess releases from upstream reservoirs]

$$- \sum_{p \epsilon P} \alpha_{py} Y_p^{11} \;-\; R_y^{11} \;=\; S_{y+1}^{11}$$

− [additional firm- and incremental secondary-yield releases] − [excess release] = [final storage]

$$S_0^{\max^{11}} \;\geq\; S_y^{11}$$

[over-year capacity] ≥ [over-year storage]

2. *Within-year storage.* For each period t and mean probability p:

$$s_{pt}^{11} \;+ \beta_{pt}^{11}\,(Y_p^{11} + v_p^{11} - v_p^6 - v_p^4)$$

[initial storage] + [additional regulated and interflow yield inflow: $v_p^{11} - v_p^6 - v_p^4 = 0.9v_p^9 - 1.6v_p^1$]

$$+\, y_{pt}^4 + y_{pt}^6 - 0.3\,(\delta_{pt}^5 T_p^5 - D_{pt}^5 + E_{pt}^5) - (\delta_{pt}^7 T_p^7 - D_{pt}^7 + E_{pt}^7)$$

+ [yields from upstream reservoirs less net consumption at sites 5 and 7]

$$-0.6\left(\sum_{p \epsilon P} T_{pt}^8\right) - 0.2\,(\delta_{pt}^{10} T_p^{10} - D_{pt}^{10} + E_{pt}^{10}) \;-\; y_{pt}^{11} \;=\; s_{p,t+1}^{11}$$

− [within-year consumption at site 8] − [within-year consumption at site 10] − [yield release] = [final storage]

3. *Equivalent flood storage capacity.* For each period t:

$$EC_t^{11} = \alpha_{11,t}\,\{F_{10,11,t}(EC_t^{10}) + SF_t^{11} + \hat{F}_{11,12,t}(K^{11} - SF_t^{11})\}$$

[equivalent capacity] = [functions of actual and equivalent flood and nonflood capacities]

4. *Total reservoir capacity.* For each period t:

$$K^{11} \;\geq s^{\text{dead}^{11}} + \; S_0^{\max^{11}} \;+\; \sum_{p \epsilon P} s_{pt}^{11} \;+\; SF_t^{11}$$

[total capacity] ≥ [dead storage] + [over-year capacity] + [within-year storage] + [flood storage capacity]

5. *Recreation.* For each period t in the recreation season:

$$\frac{1}{n}\sum_{y}^{n} S_y^{11} + S^{\text{dead}^{11}} + \frac{1}{2}\sum_{p \epsilon P} p\,(s_{pt}^{11} + s_{p,t+1}^{11}) = \;T^{11} \;-\; D_t^{11} \;+\; E_t^{11}$$

[average over-year storage] + [dead storage] + [expected within-year storage] = [target volume] − [deficit] + [surplus]

Site 12: City 2

1. *Allocation of water supply*. For each period t and mean probability p:

$$y_{pt}^{11} \geq \delta_{pt}^{12} T_p^{12} - D_{pt}^{12} + E_{pt}^{12}$$

$$\begin{bmatrix}\text{available}\\ \text{yield}\end{bmatrix} \geq \begin{bmatrix}\text{target}\\ \text{allocation}\end{bmatrix} - [\text{deficit}] + [\text{surplus}]$$

2. *Flood protection*. For each discrete flood-peak probability p in each period t:

$$X_p^{12} QN_{pt}^{12} = QR_{pt}^{12} + QS_{pt}^{12}$$

$$\begin{bmatrix}\text{unregulated peak}\\ \text{flood flow}\end{bmatrix} = \begin{bmatrix}\text{peak flow}\\ \text{contained in}\\ \text{channel}\end{bmatrix} + \begin{bmatrix}\text{peak-flow reduction}\\ \text{from upstream flood}\\ \text{storage capacity}\end{bmatrix}$$

$$EC_t^{11} \geq R_{11,12,p}\,(QS_{pt}^{12})$$

$$\begin{bmatrix}\text{equivalent flood}\\ \text{storage capacity}\end{bmatrix} \geq \begin{bmatrix}\text{function defining storage capacity at site 11}\\ \text{required to reduce peak flow at site 12}\end{bmatrix}$$

$$QR_p^{12} \geq QR_{pt}^{12}$$

$$\begin{bmatrix}\text{channel flow}\\ \text{capacity}\end{bmatrix} \geq \begin{bmatrix}\text{flow capacity required}\\ \text{in period } t\end{bmatrix}$$

$$\sum_p X_p^{12} \leq 1 \qquad X_p^{12} = \text{integer}$$

$$\begin{bmatrix}\text{indicator of optimal flood protection}\\ \text{level; only one to be selected, if any}\end{bmatrix}$$

The above set of constraints does not include any restrictions on the investments that can be made by any particular agency or group j nor any limitations on the flow at various sites in the basin. These budget or flow constraints should be specified where applicable.

Once a solution to the model is obtained, it should be examined to see if the assumed storage heads, the estimated equivalent flood storage capacities, and the piecewise linear segments of the objective or constraint functions are sufficiently accurate in the region of the solution. If not, adjustments may be required prior to resolving the model.

As a final means of model evaluation and solution improvement, the particular mix of investment alternatives and operating policies suggested by the solution of the preliminary screening model should be simulated using actual or synthetic hydrologic data. Simulation methods can be used to verify the mean probabilities of various yields and allocations, as well as to evaluate more accurately the benefits and losses resulting from those yields and allocations. For the purposes of simulation, a multiple-reservoir operating policy must be defined.[1,10] The information derived from these preliminary screening models can be used to assist in the development of these operating policies, as will be illustrated in the following example.

A Numerical Example

To illustrate in more detail a portion of the above model pertaining to the estimation of the active reservoir capacity required for firm and secondary yields, consider the basin shown in Fig. 5-10, with the exception of sites 2, 3, 7, and 10. Assume for this example that only two yields are of interest, namely, a firm yield having a 0.9 mean probability of exceedence and a secondary yield having a 0.7 mean probability of exceedence. The secondary yield is needed only for meeting a portion of the irrigation demand at site 8. The remaining irrigation demand at site 8 and the industrial and municipal demands at sites 5 and 12 must be met from firm yields. In this example, all water yield demands or targets $\delta^s_{pt} T^s_p$ at sites 5, 8, and 12 are assumed known in each period t. The objective of this analysis will be to determine the least cost combination of reservoir capacities at sites 4, 6, and 11 required to supply various percentages of the target demands at sites 5, 8, and 12.

In place of loss functions for determining the priorities for allocating demand deficits, if any, the water yield demands at each user site and in all periods t will be assumed to have the same priority. Hence, the percentages of the demand targets that can be met are to be equitably distributed among all uses throughout the basin.

This can be done by defining some unknown variables f^s_p, each denoting the fraction of the firm or incremental secondary demand target $\delta^s_{pt} T^s_p$ allocated in each period t to each site s. Equity will be achieved in this example by minimizing the differences, if any, between target allocation fractions f^s_p at each site s. Let g^s_p be the difference between the most downstream use-allocation fraction $f^{12}_{0.9}$ and any other upstream allocation fraction f^s_p. Since site 12 is downstream of all other sites, it is possible to increase $f^{12}_{0.9}$ by decreasing any or all of the other upstream allocation fractions. Without pumping water upstream, the reverse is not always possible. Hence, the differences $f^{12}_{0.9} - f^s_p = g^s_p$ are nonnegative. To ensure that these differences are minimized, they must be multiplied by a number M sufficiently larger than the sum of the annual costs $C^s(K^s)$ of each reservoir capacity K^s in the objective function:

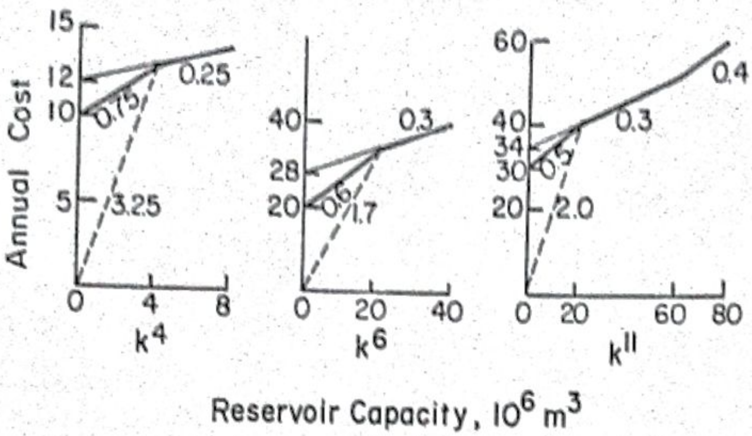
$$\text{Minimize} \quad \sum_{s=4,6,11} C^s(K^s) + M(g^5_{0.9} + g^8_{0.9} + g^8_{0.7}) \tag{E-0}$$

where

$$\begin{aligned} f^{12}_{0.9} - f^5_{0.9} - g^5_{0.9} &= 0 \\ f^{12}_{0.9} - f^8_{0.9} - g^8_{0.9} &= 0 \\ f^{12}_{0.9} - f^8_{0.7} - g^8_{0.7} &= 0 \end{aligned} \tag{E-1}$$

[In situations in which the sign of g^s_p is uncertain, the variables g^s_p in constraint equations (E-1) can be replaced by the difference of two nonnegative variables $g^s_{p1} - g^s_{p2}$, and in the objective function (E-0), each g^s_p can be replaced by the sum $g^s_{p1} + g^s_{p2}$.]

The annual cost functions $C^s(K^s)$ for each reservoir considered in this example problem are illustrated in Fig. 5-12. Also shown are the maximum capacities and the linear approximations and constraints required to define the fixed, concave-convex, cost functions for mixed integer or separable programming models.



MIXED INTEGER APPROXIMATIONS

SEPARABLE PROGRAMMING APPROXIMATIONS

Site 4. Maximum reservoir capacity = $8 \times 10^6 m^3$

Mixed integer:

$$C^4(K^4) \simeq 10z_1^4 + 0.75k_1^4 + 12z_2^4 + 0.25k_2^4$$
$$K^4 - (k_1^4 + k_2^4) = 0$$
$$k_1^4 - 4z_1^4 \leq 0$$
$$k_2^4 - 8z_2^4 \leq 0$$
$$z_1^4 + z_2^4 \leq 1$$

Separable:

$$C^4(K^4) \simeq 3.25k_1^4 + 0.25k_2^4$$
$$K^4 - (k_1^4 + k_2^4) = 0$$
$$k_1^4 \leq 4$$
$$k_2^4 \leq 8 - 4$$

Site 6. Maximum reservoir capacity = $40 \times 10^6 m^3$

Mixed integer:

$$C^6(K^6) \simeq 22z_1^6 + 0.6k_1^6 + 28z_2^6 + 0.3k_2^6$$
$$K^6 - (k_1^6 + k_2^6) = 0$$
$$k_1^6 - 20z_1^6 \leq 0$$
$$k_2^6 - 40z_2^6 \leq 0$$
$$z_1^6 + z_2^6 \leq 1$$

Separable:

$$C^6(K^6) \simeq 1.7k_1^6 + 0.3k_2^6$$
$$K^6 - (k_1^6 + k_2^6) = 0$$
$$k_1^6 \leq 20$$
$$k_2^6 \leq 40 - 20$$

Site 11. Maximum reservoir capacity = $80 \times 10^6 m^3$

Mixed integer:

$$C^{11}(K^{11}) \simeq 30z_1^{11} + 0.5k_1^{11} + 34z_2^{11} + 0.3k_2^{11} + 0.4k_2^{11}$$
$$K^{11} - (k_1^{11} + k_2^{11} + k_2^{11}) = 0$$
$$k_1^{11} - 20z_1^{11} \leq 0$$
$$k_2^{11} - 60z_2^{11} \leq 0$$
$$k_3^{11} - 20z_2^{11} \leq 0$$
$$z_1^{11} + z_2^{11} \leq 1$$

Separable:

$$C^{11}(K^{11}) \simeq 2.0k_1^{11} + 0.3k_2^{11} + 0.4k_3^{11}$$
$$K^{11} - (k_1^{11} + k_2^{11} + k_3^{11}) = 0$$
$$k_1^{11} \leq 20$$
$$k_2^{11} \leq 60 - 20$$
$$k_3^{11} \leq 80 - 60$$

FIGURE 5-12
Linear approximations of reservoir cost functions for mixed integer and separable programming models.

Tables 5-3 and 5-5 provide the remaining data needed to model this problem. Note from Table 5-5 that only two periods t are considered in this example.

The remaining constraints at each site of interest in the basin are listed below. The unknown variables are on the left side of each constraint equation, the known variables are on the right.

Table 5-5 RECORDED GAGE FLOWS AND TARGET DEMANDS FOR EXAMPLE PROBLEM. (ALL UNITS IN 10^6 m^3)

Year	Gage flows, site 1			Gage flows, site 9			Annual flow
y	$t=1$	$t=2$	Annual	$t=1$	$t=2$	Annual	Rank m
1	2.0	5.0	7.0	53	106	159	1
2	1.0	2.0	3.0	28	57	85	6
3	2.0	3.0	5.0	39	78	117	3
4	0.5	0.5	1.0	17	35	52	9
5	1.0	1.0	2.0	24	48	72	8
6	1.0	4.0	5.0	37	73	110	4
7	2.0	4.0	6.0	46	91	137	2
8	1.5	1.5	3.0	27	53	80	7
9	1.0	3.0	4.0	32	63	95	5
Average ($m \le 9$)	1.3	2.7	4.0	33.6	67.1	100.7	
Average ($m \le 7$)	1.5	3.2	4.7	37.4	74.4	111.8	

Site	Water target demands $\delta^s_{pt} T^s_p$			
	$p = 0.9$		$p = 0.7$	
s	$t=1$	$t=2$	$t=1$	$t=2$
5	2.0	2.0	—	—
8	20.0	—	20.0	—
12	50.0	30.0	—	—

($T^8_{0.7}$ is an incremental demand.)

1. *Sites 4 and 6*: Over-year and within-year reservoir storage capacities:

$$S^4_{y+1} - S^4_y + \sum_{p \in P_1} \alpha_{yp} Y^4_p + R^4_y = 1.6G^1_y - [U^4_y = 1.6\,(1.0)] \qquad \forall y \qquad \text{(E-2)}$$

$$S^6_{y+1} - S^6_y + \sum_{p \in P_2} \alpha_{yp} Y^6_p + R^6_y = 0.4G^9_y - [U^6_y = \begin{cases} (0.4)\,(80)] & \forall y \neq 4,5 \\ (0.4)\,(52)] & y = 4,5 \end{cases} \qquad \text{(E-3)}$$

Note that, at reservoir site 4, a yield having a mean probability of exceedence of 0.9 is the only yield of interest, since there are no secondary demands downstream. Yields having mean probabilities of exceedence of 0.9 and 0.7 are demanded downstream of site 6. Hence, the sets P of probabilities p of interest are not the same at both reservoir sites and are, therefore, designated P_1 and P_2, respectively. At both reservoir sites, the variable α_{yp} is 1 whenever p equals 0.9 and is 1 at site 6 when p is 0.7, unless $y = 4$ or 5. Also, recall that if $y = 9$, then $y + 1 = 1$:

$$S^4_y - S^{\max 4}_0 \le 0 \qquad \forall y \qquad \text{(E-4)}$$

$$S^6_y - S^{\max 6}_0 \le 0 \qquad \forall y \qquad \text{(E-5)}$$

$$s^4_{0.9,t+1} - s^4_{0.9t} - \beta^4_{0.9t}Y^4_{0.9} + y^4_{0.9t} = \beta^4_{0.9t}v^4_{0.9} = \begin{cases} \dfrac{1.3}{4.0}(1.6)(1.0) & t=1 \\ \dfrac{2.7}{4.0}(1.6)(1.0) & t=2 \end{cases} \tag{E-6}$$

$$s^6_{0.9,t+1} - s^6_{0.9t} - \beta^6_{0.9t}Y^6_{0.9} + y^6_{0.9t} = \beta^6_{0.9t}v^6_{0.9} = \begin{cases} \dfrac{33.6}{100.7}(0.4)(52) & t=1 \\ \dfrac{67.1}{100.7}(0.4)(52) & t=2 \end{cases} \tag{E-7}$$

$$s^6_{0.7,t+1} - s^6_{0.7t} - \beta^6_{0.7t}Y^6_{0.7} + y^6_{0.7t} = \beta^6_{0.7t}v^6_{0.7} = \begin{cases} \dfrac{37.4}{111.8}(0.4)(80-52) & t=1 \\ \dfrac{74.4}{111.8}(0.4)(80-52) & t=2 \end{cases} \tag{E-8}$$

$$K^4 - S_0^{\max^4} - s^4_{0.9t} \geq 0 \qquad \forall t \tag{E-9}$$

$$K^6 - S_0^{\max^6} - s^6_{0.9t} - s^6_{0.7t} \geq 0 \qquad \forall t \tag{E-10}$$

2. *Site 5*: Firm-yield allocation:

$$f^5_{0.9}\delta^5_{0.9t}T^5_{0.9} - y^4_{0.9t} \leq \beta^5_{0.9t}(v^5_{0.9} - v^4_{0.9}) = \begin{cases} \dfrac{1.3}{4.0}(1.8-1.6)(1.0) & t=1 \\ \dfrac{2.7}{4.0}(1.8-1.6)(1.0) & t=2 \end{cases} \tag{E-11}$$

3. *Site 8*: Firm- and secondary-yield allocations in period $t = 1$ only:

$$f^8_{0.9}\delta^8_{0.9}T^8_{0.9} - Y^6_{0.9,1} \leq \beta^8_{0.9,1}(v^8_{0.9} - v^6_{0.9}) = \frac{33.6}{100.7}(0.6-0.4)(52) \tag{E-12}$$

$$f^8_{0.7}\delta^8_{0.7,1}T^8_{0.7} - Y^6_{0.7,1} \leq \beta^8_{0.7,1}(v^8_{0.7} - v^6_{0.7}) = \frac{37.4}{111.8}(0.6-0.4)(80-52) \tag{E-13}$$

4. *Site 11*: Over-year and within-year reservoir storage capacities required to increase or redistribute firm yields:

$$S^{11}_{y+1} - S^{11}_y - R^4_y - R^6_y - \alpha_{y0.7}\left(Y^6_{0.7} - 0.6\sum_t f^8_{0.7}\delta^8_{0.7t}T^8_{0.7}\right) + Y^{11}_{0.9} + R^{11}_y$$

$$= (1.3-0.4)\,G^9_y - 1.6G^1_y - (U^{11}_y - U^4_y - U^6_y)$$

$$= (1.3-0.4)(G^9_y) - 1.6G^1_y - (1.3)(52) + 1.6(1.0) + 0.4(52) \qquad \forall y \tag{E-14}$$

Note that the net incremental secondary yields available at site 11 are included in the inflow term of these over-year constraints, permitting the conversion of these upstream secondary incremental yields to firm yields downstream from site 11.

$$S^{11}_y - S_0^{\max^{11}} \leq 0 \qquad \forall y \tag{E-15}$$

$$s^{11}_{0.9,t+1} - s^{11}_{0.9t} - \beta^{11}_{0.9t}Y^{11}_{0.9} - y^4_{0.9t} - y^6_{0.9t} + 0.3f^5_{0.9}\delta^5_{0.9t}T^5_{0.9}$$
$$+ 0.6f^8_{0.9}\delta^8_{0.9t}T^8_{0.9} + y^{11}_{0.9t} = \beta^{11}_{0.9t}(v^{11}_{0.9} - v^6_{0.9} - v^4_{0.9})$$

$$= \begin{cases} \frac{33.6}{100.7}\,[(1.3-0.4)\,(52)-1.6(1.0)] & t=1 \\ \frac{67.1}{100.7}\,[(1.3-0.4)\,(52)-1.6(1.0)] & t=2 \end{cases} \qquad \text{(E-16)}$$

$$K^{11} - S_0^{\max^{11}} - s_{0.9t}^{11} \geq 0 \qquad \forall t \qquad \text{(E-17)}$$

5. *Site 12*: Allocation of firm yield from site 11:

$$y_{0.9t}^{11} - f_{0.9}^{12}\delta_{0.9t}^{12}T_{0.9}^{12} \geq 0 \qquad \forall t \qquad \text{(E-18)}$$

6. Fraction of target allocation required at site 12 in each period t:

$$f_{0.9}^{12} = \text{various values} > 0 \qquad \text{(E-19)}$$

Excluding the nonnegativity restrictions on the decision variables, and the integer restrictions on the z_i^s variables, the 13 constraints shown in Fig. 5-12 and the 78 constraints [(E-1) through (E-19)], together with the objective function (E-0), complete the 9-year, two-period model of sites 4, 5, 6, 8, 11, and 12. The model as developed does not take advantage of the grouping of data as discussed on page 172. If it did, the 91 constraints listed above could be reduced to 64 by (1) combining years 6 and 7 and years 8 and 9, thereby defining only seven constraints (E-2), (E-3), and (E-14); (2) defining the over-year capacity constraints (E-4), (E-5), and (E-15) for only years 2, 4, and 8; and (3) defining the capacity constraints (E-9), (E-10), and (E-17) for only period $t = 1$. The latter two types of constraint reductions (2 and 3) apply in any situation in which it is obvious after an examination of the sequence of unregulated flows that certain over-year and within-year reservoir capacity constraints will be satisfied even if they are not included within the model.

Solutions

The standard mixed integer and separable programming (MPSX/MIP) algorithms available at most IBM computing facilities were used to solve this example problem. Since the integer programming model is more accurate than the separable model with respect to the fixed cost portion of the reservoir cost functions, only the integer programming results are summarized in Table 5-6 and Fig. 5-13.

Note from Table 5-6 that, as the allocation fraction $f_{0.9}^{12}$ of the firm-yield target demand at site 12 is increased, so are those allocation fractions at the other use sites in the basin, at least up to the full allocation of each firm- and secondary-yield target. As the fractions f_p^s increase together to 1.0, trade-offs are made among the reservoir capacities at each site in order to achieve a least-cost combination of capacities.

When $f_{0.9}^{12} > 1.0$, it is impossible to maintain equity, i.e., to keep all target allocation fractions equal. This results from either insufficient streamflow or insufficient reservoir capacity, or both. At $f_{0.9}^{12} = 1.52$, all reservoirs are at their full capacity and no water is allocated to the uses at upstream sites 5 and 8. In

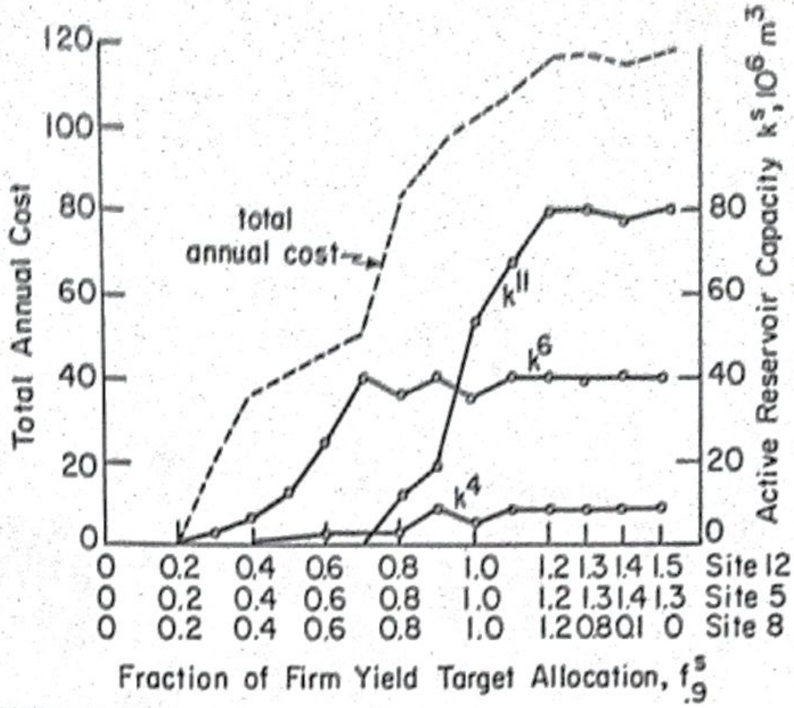


FIGURE 5-13
Minimum total annual cost and corresponding reservoir capacities required for various fractions of target allocations.

this case, as when $f_{0.9}^{12} = 1.2$ and 1.3, reservoir capacity is limiting, and not all the water released from reservoir 11 can be used. Conversely, when $f_{0.9}^{12} = 1.4$ and 1.5 streamflows are limiting, and all reservoir releases at site 11 are allocated to the use at site 12. There are insufficient streamflows and capacity to be able to increase the target allocation fraction $f_{0.9}^{12}$, at site 12, beyond 1.52.

Note from Table 5-6 that, as the fraction allocated to site 12 increases beyond 1.0, the allocations at site 8 begin to decrease much more than the allocation fraction at site 5. This is, in part, due to the relatively small quantity of water available in the tributary supplying site 5 compared to the main river which supplies site 8. Reducing the allocation at site 5 does not increase the water available at site 12 as much as do similar reductions in the firm or secondary target allocation fractions at site 8.

For all target allocation fractions equal to 0.9, both integer and separable programming solutions are compared in somewhat more detail in Table 5-7. In this particular example, there are at least two equally optimal separable programming solutions, one of which was identical to the second best integer programming solution. The fact that the difference in the total annual cost between the best and second best solutions is small illustrates a common characteristic of most river basin development problems, namely, that the total investment cost does not change markedly for various alternative investment policies near the optimum. This example also illustrates the fact that separable programming may yield only near-optimal but not globally optimal solutions.

Table 5-6 INTEGER PROGRAMMING SOLUTIONS OF EXAMPLE PROBLEM FOR VARIOUS VALUES OF $f^{12}_{0.9}$ FROM 0.20 TO 1.52

Use site s	Fraction of target allocated f^s_p	Alternative solutions														
12	$f^{12}_{0.9}$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.52
5	$f^{5}_{0.9}$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.3	0
8	$f^{8}_{0.9}$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	0.8	0.1	0	0
8	$f^{8}_{0.7}$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.0	0.8	0.7	0.7	0	0
Reservoir site c	**Capacity** K^c															
4	K^4	0	0	0.2	0.5	1.0	2.3	1.9	8.0	3.5	8.0	8.0	8.0	8.0	8.0	8.0
6	K^6	0	0.4	4.6	12.6	21.6	40.0	34.7	40.0	35.5	40.0	40.0	40.0	40.0	40.0	40.0
11	K^{11}	0	0	0	0	0	0	10.8	17.4	51.6	68.9	80.0	80.0	77.4	79.9	80.0

The analysis of the implications of these solutions could continue, or the problem could be restructured to answer other questions, such as what target allocations can be obtained from special combinations of reservoir capacities. Rather than do this, a single question will be examined, namely, how good are these estimates of reservoir capacities needed to make specified firm- and secondary-yield allocations? To attempt to answer this, the information contained in any particular solution must be translated into a language more easily understood by those responsible for operating river basin systems. One important step is the construction of reservoir rule curves (i.e., release rules based on current storage volumes) for each reservoir. Next, a policy must be defined that coordinates the operation of a system of reservoirs and water users. Once this is done, the river basin plan can be simulated to evaluate and improve any proposed investment policy with respect to its ability to supply the desired yields with the desired reliability to each water user.

Reservoir Rule Curves

To illustrate the construction of reservoir rule curves, the partial solution from the integer programming model shown in Table 5-7 will be used. Based on the annual firm yields of 3.4, 21.0, and 72.0 at reservoir sites 4, 6, and 11, respec-

Table 5-7 COMPARISON OF OPTIMAL INTEGER AND SEPARABLE PROGRAMMING SOLUTIONS FOR 90 PERCENT TARGET ALLOCATIONS

Parameter	Integer programming solution			Separable programming solutions					
				Solution 1			Solution 2		
Total annual cost units		92.7				92.8 (89.2*)			
Reservoir site s	4	6	11	4	6	11	4	6	11
Over-year capacity $S_0^{\max s}$	7.4	15.4	3.5	2.0	9.8	14.4	2.0	15.4	8.9
Total capacity K^s	8.0	40.0	17.4	2.6	28.1	34.6	2.6	40.0	22.8
Firm yield, period 1, $y_{0.9,1}^s$	1.7	21.0	45.0	1.7	14.5	45.0	1.7	20.8	45.0
Firm yield, period 2, $y_{0.9,2}^s$	1.7	0	27.0	1.7	6.3	27.0	1.7	0	27.0
Secondary yield, period 1, $y_{0.7,1}^s$	—	16.0	—	—	16.1	—	—	16.1	—
Within-year storage, period 1, $s_{0.9,1}^s$	0.6	13.9	13.9	0.6	7.6	20.1	0.6	13.9	13.9
Within-year storage, period 2, $s_{0.9,2}^s$	0	0	0	0	0	0	0	0	0
Within-year storage, period 1, $s_{0.7,1}^s$	—	10.7	—	—	10.7	—	—	10.7	—
Within-year storage, period 2, $s_{0.7,2}^s$	—	0	—	—	0	—	—	0	—

*Total annual cost based on linear approximation of fixed costs as illustrated in Fig. 5-12.

tively, Eq. (5-6) can be used to compute the over-year storage required for those firm yields at those sites. (The variable S_{ij}^{maxs} is not an adequate estimate of the over-year storage required for any particular firm yield in multireservoir systems or when secondary incremental yields are defined, since it includes storage requirements for all yields and for possible regulation of excess releases R_{ij}^{t}.) To the over-year storage can be added the within-year storage requirements s_{pt}^{s} for the firm yield in each period t. When the actual reservoir storage is less than this sum at any time within the year, only firm-yield releases should be made from the reservoir.

Next, the over-year storage requirements for each successive incremental secondary yield Y_p can be obtained from Eq. (5-7). This, plus the within-year requirements s_{pt}^{s} for those respective secondary yields, defines the reservoir storage zone from which releases should be made for only those downstream yield demands requiring a reliability equal to or greater than p.

These storage requirements for each successive yield can be plotted in the form of rule curves. Using the information in Table 5-7 and Eqs. (5-6) and (5-7), rule curves constructed for the reservoirs at sites 4, 6, and 11 are illustrated in Fig. 5-14. Since only firm-yield targets are defined downstream of the reservoirs at sites 4 and 11, all releases from these reservoirs will be for firm yields only (i.e., $p = 0.9$). Both firm and secondary ($p = 0.7$) releases will be made from the reservoir at site 6.

For a more detailed explanation of rule-curve construction, consider the reservoir at site 6. To obtain a firm yield of 21×10^6 m³, Eq. (5-6) indicates that an over-year storage of 0.2×10^6 m³ is required. To this is added the within-year storage (from Table 5-7) of 13.9 at the beginning of period 1 and 0 at the beginning of period 2. This then defines the zone in which only firm releases should be made if the actual storage is within this zone.

From Eq. (5-7) the total over-year capacity of 5.0×10^6 m³ is required for a total secondary release of 37, 21 of which is firm. Hence, $5.0 - 0.2$ or 4.8×10^6 m³ of over-year storage is required for the incremental secondary yield of 16×10^6 m³. To this is added 10.7×10^6 m³ (from Table 5-7) of within-year storage needed at the beginning of period 1. Again, no within-year storage is needed at the beginning of period 2. This secondary-yield storage requirement is then added to the firm-yield requirement, and it defines the zone in which firm and secondary releases are permitted. If the actual storage volume exceeds this sum at any time, then any release can be made, as desired. In this example, of course, only firm- and secondary-yield targets are defined; hence, all releases will be for satisfying only these targets.

To define the rule curves of reservoirs whose inflows are partially regulated, such as at site 11, those partially regulated inflows obtained from the solution of the model must be used in Eqs. (5-6) or (5-7). If flood storage capacity is included in the model, reservoir rule curves would also define those zones in which stored water would have to be released, even if not needed downstream, in order to maintain flood storage capacity.

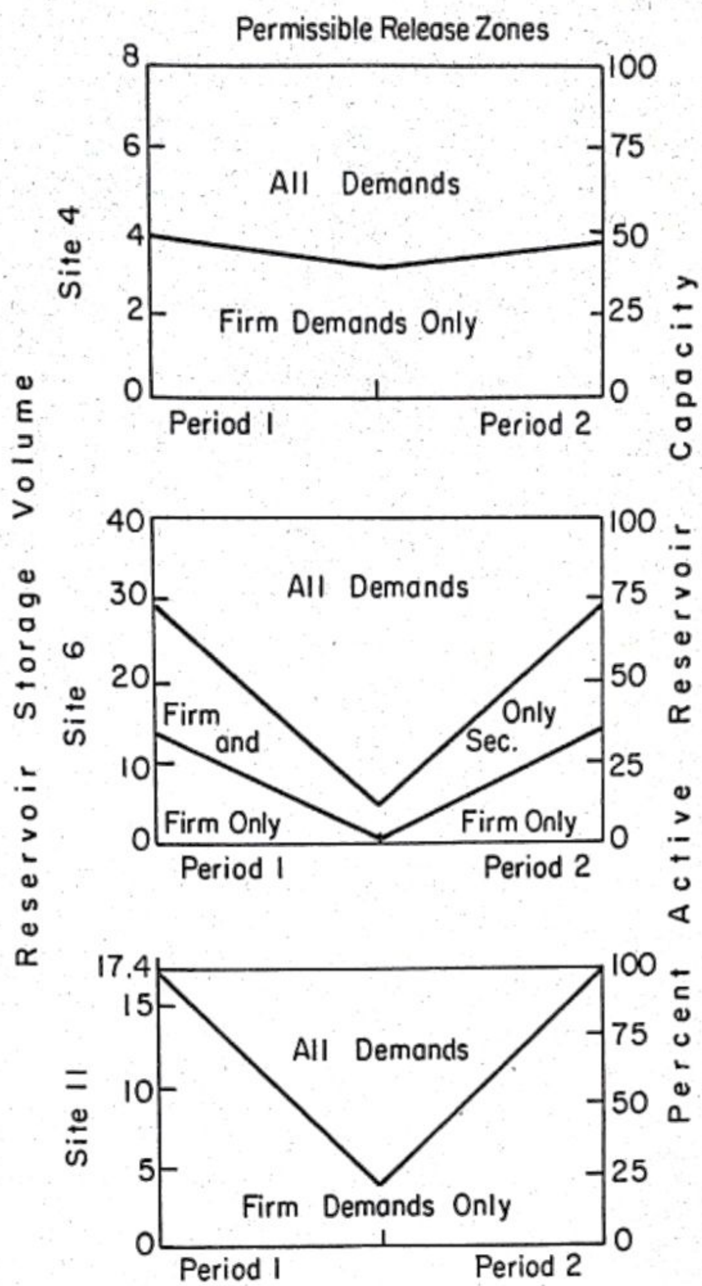


FIGURE 5-14
Reservoir rule curves indicating allowable release as a function of current storage volume.

Simulation of Results

To evaluate adequately the capability of any set of reservoir capacities and operating policies to achieve desired yield allocations and reliabilities would require a detailed simulation, preferably using streamflow synthesis techniques such as those discussed in Chap. 3 and in Ref. 23. However to illustrate the use of simulation, the historic flows (Table 5-5) will be used as a first ap-

Table 5-8 SUMMARY OF STEADY-STATE SIMULATION RESULTS FOR HISTORICAL STREAMFLOWS*

		Reservoir, site 4			Allocation at site 5	Reservoir, site 6			Allocation at site 8		Reservoir, site 11			Allocation at site 12
Year	Period	Initial storage	Inflow†	Release	Firm	Initial storage	Inflow‡	Release	Firm	Secondary	Initial storage	Inflow¶	Release	Firm
1	1	8.0	3.2	3.2	1.8	40.0	21.2	25.4	18	18	17.4	51.0	51.0	45
	2	8.0	8.0	8.0	1.8	35.8	42.4	38.2	0	0	17.4	133.1	133.1	27
2	1	8.0	1.6	1.6	1.8	40.0	11.2	30.4	18	18	17.4	33.5	45.0	45
	2	8.0	3.2	3.2	1.8	20.8	22.8	3.6	0	0	5.9	54.4	42.9	27
3	1	8.0	3.2	3.2	1.8	40.0	15.6	28.2	18	18	17.4	41.2	45.0	45
	2	8.0	4.8	4.8	1.8	27.4	31.2	18.6	0	0	13.6	88.3	84.4	27
4	1	8.0	0.8	2.6	1.8	40.0	6.8	32.6	18	18	17.4	27.6	45.0	45
	2	6.2	0.8	1.7	1.8	14.2	14.0	0	0	0	0	31.9	27.0	27
5	1	5.3	1.6	3.7	1.8	28.2	9.6	37.6	18	16.4	4.9	40.1	45.0	45
	2	3.2	1.6	1.6	1.8	0.2	19.2	0	0	0	0	42.7	27.0	27
6	1	3.2	1.6	1.6	1.8	19.4	14.8	28.6	18	18	15.7	39.8	45.0	45
	2	3.2	6.4	1.6	1.8	5.6	29.2	0	0	0	10.4	60.4	53.4	27
7	1	8.0	3.2	3.2	1.8	34.8	18.4	26.8	18	18	17.4	46.1	46.1	45
	2	8.0	6.4	6.4	1.8	26.4	36.4	22.8	0	0	17.4	104.2	104.2	27
8	1	8.0	2.4	2.4	1.8	40.0	10.8	30.6	18	18	17.4	32.8	45.0	45
	2	8.0	2.4	2.4	1.8	20.2	21.2	1.4	0	0	5.2	48.6	36.3	27
9	1	8.0	1.6	1.6	1.8	40.0	12.8	29.6	18	18	17.4	36.3	45.0	45
	2	8.0	4.8	4.8	1.8	23.2	25.2	8.4	0	0	8.7	64.6	55.8	27

*All values are rounded to nearest 0.1×10^6 m³. Calculations were performed to the nearest 0.01×10^6 m³.

†1.6 times the unregulated flow at gage site 1, $G_{\nu t}^{1}$.

‡0.4 times the unregulated flow at gage site 9, $G_{\nu t}^{9}$.

¶Release at 4 + release at 6 $+ (1.3 - 0.4)G_{\nu t}^{9} - (1.6)G_{\nu t}^{1} - 0.3$ (allocation at 5) − 0.6 (allocation at 8).

proximate evaluation of the integer programming solution in Table 5-7. This will require the data in Table 5-3, the reservoir rule curves illustrated in Fig. 5-14, and a policy which coordinates the operation of the three reservoirs.

The reservoir operating policy will be as follows: Releases will be made from reservoirs at sites 4, 6, and 11 to meet only the immediate downstream target demands (90 percent of those stated in Table 5-5) at sites 5, 8, and 12, respectively, subject to the permissible releases as defined by the reservoir rule curves. If water must be released from reservoirs 4 and 6 to satisfy the demand at 12, the releases from reservoirs 4 and 6 will be the minimum required to meet the demand deficit at 12. They will also be made so as to equalize, if possible, the percentage of remaining storage in each of the release zones of these two upstream reservoirs.

While this policy seems reasonable, it is probably not optimal. Hence, if any adjustment is to be made in reservoir capacity, it is likely to require some increase in capacity to meet the desired quantity and reliability of the target demands.

Table 5-8 summarizes the steady-state simulation of the historical flows of the three-reservoir, three-use system. In this simulation, each of the three firm-yield targets can be satisfied in each period of the 9 years of record. Hence, the firm-yield allocations appear to be at least 90 percent reliable. The secondary-yield allocations at site 8 are met in all but one year; hence, their reliability is at least the desired 70 percent, and perhaps even 80 percent. In the period when the secondary-yield target cannot be satisfied, the deficit is only 1.6×10^6 m^3, or about 9 percent of the 18×10^6 m^3 target demand.

The 17.4×10^6 m^3 of reservoir capacity at site 11 is sufficient to meet the firm demands at site 12 except in years 4 and 5. In period 1 of year 4, an additional 0.94×10^6 m^3 is needed and can be withdrawn from the unrestricted release zone of reservoir 4. In year 5, an additional 9.48×10^6 m^3 is needed in period 1 at site 12. Of this, 2.1 can come from the unrestricted release zone of reservoir 4, and the remaining 7.38 can be released from the unrestricted and "firm and secondary" release zones of reservoir 6.

The water stored in the firm release zones of reservoirs 4 and 6 is not needed in this simulation of historical flows. This fact, plus the additional reliability of the secondary yields, suggests that the approximations and assumptions made in this model may lead to a conservative estimate of the required active reservoir capacities, at least for this example problem. Whether or not this is true can be verified only by a more detailed simulation analysis.

SUMMARY

There are a number of techniques available for defining and evaluating various surface-water management and investment alternatives. Many of these techniques, such as mass diagram and storage-yield analyses, are most appropriately applied to single reservoir systems. What has been attempted here is the development of some rather simple static optimization models that can complement some of the more traditionally used procedures for the preliminary screening of regional or basin-wide surface-water management plans or policies. The models are deterministic in size and character, yet estimates are available of the reliability of each flow, yield, or allocation. The models also explicitly consider the requirements, if any, for over-year, within-year, and flood-control storage capacity in multiple-purpose reservoirs. Such information is needed to develop rule curves for defining reservoir operating policies.

One objective in the development of the models presented in this chapter has been to minimize the number of constraints required to adequately describe a complex river basin system. Many of the assumptions made in order to reduce model size are admittedly approximations of reality. In addition, there are always many aspects of river basin investments and management that are not easily quantified and included in the model. Therefore, as with any preliminary screening tool, the solutions of these models should be further evaluated and improved, using more precise simulation techniques. An example problem has been used to illustrate how the solutions of the preliminary screening models can be simulated to obtain better estimates of system performance. Again, the purpose of any screening model is not necessarily to find the best investment and operating policy, but to provide a good start for a more detailed, more accurate, and no doubt more expensive analysis which will hopefully lead to the best policy.

There are many possible extensions and modifications to the modeling methodology presented in this chapter. Any one model will never be the most appropriate one for analyzing all surface-water management problems in all river basins. Rather than review many of the possible extensions or modifications that could be made, what has been emphasized is a general formulation or a general modeling approach. This basic framework can be modified or extended as appropriate for any specific river basin planning problem.

Developing models of river basin systems is an art. There is no single best way to do it, although admittedly for specific water-resources problems some approaches have proven better than others. The only way this writer knows to develop and improve one's mastery of this art is to study the fundamentals of modeling and then to practice and compare. The fundamentals discussed in this chapter should provide a basis for model comparison and for structuring preliminary screening models of more complex water-resources investment and operating policy problems. This writer's experience suggests that if management models are structured by competent analysts who (1) are fully acquainted with the specific water-management problem, as well as with the art and science of model development and solution, and (2) are directly involved in the planning or decision-making process, then there is an excellent chance that the information derived from the models will prove very useful to the overall planning effort.

ACKNOWLEDGMENTS

The research reported in this chapter was supported in part from funds provided by the Office of Water Research and Technology, U.S. Department of the Interior, as authorized by the Water Resources Research Act of 1964, as amended.

The writer is very grateful to Dean C. Pappus, Carl O. Foley, and the Planning Associates class of 1970–71 of the U.S. Army Corps of Engineers, who provided the original motivation for many of the ideas presented in this chapter. Several of the modeling procedures discussed for flood protection and damage reduction resulted from discussions with Warren Viessman, Jr., Gary L. Lewis, and Isaac Yomtovian, who applied these models to a Level B river basin study in Nebraska.

These modeling techniques have also been used by Hydrotechnic, Inc., to evaluate water resource investment policies in northern Algeria and applied to portions of the Upper Mures River basin in Romania by Ioan Dima, Andrei Filotti, and Alexandru Simon of the Research and Design Institute for Water Resources Engineering in Bucharest. Their experience, comments, and ideas have been extremely valuable.

Numerous individuals commented on earlier drafts of this chapter. Particularly helpful were the criticisms and suggestions of Janusz Kindler, William K. Johnson, Yves Mechin, Uri Shamir, Ying-Ming Su, John F. Wall, and Jing-Yea Yang. Of course, all opinions contained herein are the writer's and are not necessarily those of other individuals or institutions.

REFERENCES

1. HUFSCHMIDT, M. M., and M. B FIERING, "Simulation Techniques for Design of Water Resource Systems," Harvard University Press, Cambridge, Mass., 1966.
2. LANGBEIN, W. B., Plotting Positions in Frequency Analysis, in "Flood-Frequency Analyses, Manual of Hydrology, Part 3, Flood Flow Techniques," Geological Survey Water Supply Paper 154-A, p. 48, 1960.
3. RIPPL, W., The Capacity of Storage Reservoirs for Water Supply, *Proc. Inst. Civ. Eng.* (*Brit.*), vol. 71, 1883.
4. THOMAS, H. A., JR., and M. B FIERING, "Operations Research in Water Quality Management," chap. 1, pp. 1–17, Harvard Water Resources Group, Cambridge, Mass., 1963.
5. LINSLEY, R. K., and J. B. FRANZINI, "Water Resources Engineering," chap. 20, p. 578, McGraw-Hill Book Company, New York, 1964.
6. DELUCIA, R. J., "Operating Policies for Irrigation Systems Under Stochastic Regimes," Harvard Water Resources Group, Harvard University, Cambridge, Mass., 1969.
7. DUDLEY, N. J., D. T. HOWELL, and W. F. MUSGRAVE, Optimal Intraseasonal Irrigation, *Water Resour. Res.*, vol. 7, no. 4, 1971.

8. DUDLEY, N. J., D. T. HOWELL, and W. F. MUSGRAVE, Irrigation Planning 2: Choosing Optimal Acreages within a Season, *Water Resour. Res.*, vol. 7, no. 5, 1971.

9. ROGERS, P., and D. V. SMITH, "An Algorithm for Irrigation Project Planning," *ICID Bull.*, International Commission of Irrigation and Drainage, vol. 46, 1970.

10. BOWER, B. T., M. M. HUFSCHMIDT, and W. W. REEDY, Operating Procedures: Their Role in the Design of Water-Resource Systems by Simulation Analyses, in A. Maass et al., "Design of Water Resource Systems," chap. 11, p. 443, Harvard University Press, Cambridge, Mass., 1962.

11. DORFMAN, R., H. D. JACOBY, and H. A. THOMAS, JR., "Models for Managing Regional Water Quality," Harvard University Press, Cambridge, Mass., 1973.

12. DENEUFVILLE, R., and D. H. MARKS, "Systems Planning and Design: Case Studies in Modelling, Optimization and Evaluation," Prentice-Hall, Inc., Englewood Cliffs, N.J., 1974.

13. COHON, J. L., and D. H. MARKS, A Review and Evaluation of Multiobjective Programing Techniques, *Water Resour. Res.*, vol. 11, no. 2, pp. 208–220, 1975.

14. MACCRIMMON, K. R., Decision Making among Multiple-Attribute Alternatives: A Survey and Consolidated Approach, Memorandum RM–4823–ARPA, The Rand Corporation, Santa Monica, California, December 1968.

15. MAJOR, D. D., Benefit-Cost Ratios for Projects in Multiple Objective Investment Programs, *Water Resour. Res.*, vol. 5, no. 6, 1969.

16. MARGLIN, S., Role of National Planning in Project Formulation and Evaluation, in Dasgupta, Sen, and Marglin, "Guidelines for Project Evaluation," chap. II, United Nations Publication, 1972.

17. ROY, B., Problems and Methods with Multiple Objective Function, *Math. Program.*, vol. 1, 1971.

18. DORFMAN, R., Formal Models in the Design of Water Resource Systems, *Water Resour. Res.*, vol. 1, no. 3, 1965.

19. JACOBY, H. D., and D. P. LOUCKS, Combined Use of Optimization and Simulation Models in River Basin Planning, *Water Resour. Res.*, vol. 8, no. 6, 1972.

20. LECLERC, G., and D. H. MARKS, Determination of the Discharge Policy for Existing Reservoir Networks, in R. deNeufville and D. H. Marks, "Systems Planning and Design: Case Studies in Modelling, Optimization and Evaluation," Prentice-Hall, Inc., Englewood Cliffs, N.J., 1974.

21. GABLINGER, M., and D. P. LOUCKS, Markov Models for Flow Regulation, *J. Hydraul. Div., Am. Soc. Civ. Eng.*, vol. 96, no. HY1, January 1970.

22. YOUNG, G. K., JR., Finding Reservoir Operating Rules, *J. Hydraul. Div., Am. Soc. Civ. Eng.*, vol. 93, no. HY6, November 1967.

23. FIERING, M. B, and B. B. JACKSON, Synthetic Streamflows, American Geophysical Union Water Resources Monograph 1, Washington, D.C., 1971.